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ABSTRACT

Principal-agent models take outside options, determining participation and incentive constraints, as given. We construct a general equilibrium model where workers' reservation wages and the maximum punishment acceptable before workers quit are instead determined endogenously. We simultaneously extend the standard effort efficiency-wage model by incorporating noisy signals, labor market frictions, and the possibility of performance-based pay, analyzing the equilibrium response to an adverse signal, and establishing conditions under which equilibrium entails lowering wages (performance contracting) rather than firing. We provide a complete analysis of the general equilibrium comparative statics, showing, for instance, that frictions (sand-in-the-wheels) may decrease unemployment and that the equilibrium is determined by two simple aggregates which depend on the parameters of the economy, interpretable as the intercept and slope of a pseudo-labor supply curve, embedding all the binding constraints (e.g., the no-shirking and labor market participation constraints). We also show that there may exist only a firing equilibrium, a no-firing equilibrium, multiple (firing and no-firing) equilibria, and no pure-strategy equilibrium. The economy is, in general, not efficient either in the selection of the form of equilibrium or the wages paid within any type of equilibrium. We discuss welfare enhancing government interventions, including publicly provided sand-in-the-wheels.

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I. Introduction

We provide a simple general equilibrium model of labor contracting with moral hazard, in which the equilibrium wage pattern is endogenous and entails punishment for poor performance. The paper has several motivations. The first is related to the literature on agency models, and more particularly on principal-agent models with moral hazard. Standard models[4] usually posit a reservation level of utility for the agent, which corresponds to the agent's "outside option" and is supposed to be exogenous to the situation under scrutiny. Here, we endogenize this reservation level using a general competitive equilibrium approach, in which the agent's outside option corresponds to what he can get from dealing with other principals—which is solved for simultaneously within the system.

Moreover, in the standard model, to elicit effort, pay is made contingent on observed output, which is a noisy signal of the individual's effort. The literature typically doesn't ask: will the worker accept the punishment of low pay with bad outcomes? Again, there are endogenous outside opportunities. The individual can walk—at a cost—and these outside options too need to be solved for within the model. The firm needs to decide whether to limit its wage reduction (even with very bad outcomes) to the level that will just induce the individual to quit or to lower the wage further—an induced quit, the equivalent of a fire.

The paper can also be seen as an extension to the standard effort efficiency wage theory, which explains wage rigidity and unemployment through the need to provide incentives in the presence of imperfect monitoring (see for instance Shapiro-Stiglitz, 1984).[5] Standard efficiency

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[4] See, e.g., Grossman-Hart (1983), Stiglitz (1974a, 1975), and Shapiro-Stiglitz (1984).

[5] Henceforth Shapiro-Stiglitz. Earlier, MacLeod and Malcolmson (1998) argued that performance pay, which we explore here, is a more effective motivator of effort than efficiency wages.

However, wage differences among workers engaged in similar work may have adverse effects on morale and effort; and this may be even true if the differences are associated with differences in the performance of the workers' work unit, especially if (part of) the reason for those differences is perceived by the worker not to be related to worker performance. In this paper, we do not consider such effects. But see Card *et al* (2012) and Dube *et al* (2019). Breza *et al* (2018) provide causal evidence for the morale effects of pay inequality using a randomized control trial. Their results suggest that so long as pay differentials are related to performance differences, there is no adverse effect. Whether that is so when, as here, performance differences may be strongly determined by luck and where pay differences may be incommensurate with differences in effort remains a question. The analysis below ignores possible morale effects.

wage models[6] assume that wages cannot be related, either directly or indirectly, to effort (that is, a worker's contribution to the firm cannot be observed or verified by outside parties). However, in practice, the firm can receive noisy signals about a worker's effort and lower wages (or fire the worker) in response to an adverse signal. But as Shapiro and Stiglitz noted, firing or lowering wages induces incentives not to shirk only if there is some kind of friction in the labor market, which can be exogenous (switching costs) or endogenous (unemployment).[7,8] Shapiro-Stiglitz analyzed unemployment in a model with (random) supervision in which there was no signal of effort. This paper analyzes equilibrium in the alternative case, where there is such a signal, and firms can and do resort to incentives based on this signal. (As in the other polar case, here we assume no direct observations on effort.) Now, the firm can respond to the negative signal either by firing the worker or lowering his wages. In some circumstances, it may be preferable to lower the wage. Market equilibrium will still, in general, be characterized by unemployment. The unemployment enables the imposition of the low wage punishment. Without unemployment, workers would walk. *Unemployment is needed here to make credible a threat of a reduction in their wages*.[9]

Here, we model the case where both firms and workers may face costs associated with leaving or joining a firm. By contrast, in Shapiro-Stiglitz, the only labor market "friction" is

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[6] We refer here to efficiency wage models built on moral hazard arguments (worker can exert more or less effort). There are several alternative bases of efficiency wages (Stiglitz, (1969a)), including labor turnover (Stiglitz (1974b)), adverse selection (Stiglitz (1992, 1982), Weiss (1980), Nalebuff *et al* (2013), Meyer (1987)), and morale (Stiglitz (1969a, 1974c)), Akerlof and Yellen (1990)). See Stiglitz (1987) for an overview.

[7] Even in early discussions of asymmetric information (Stiglitz, 1974a), it was realized that there were two ways to address the resulting incentives problem. One was to supervise workers, which was costly, and punish (e.g. by firing) those who were detected shirking. The other was to base compensation on observed output though that was a noisy signal of effort, and doing so forced workers to bear risk which might be better borne by, say, less risk averse employers. Such incentive schemes still might be preferable to costly supervision. Holmstrom (1979) provides a general model integrating both kinds of signals. Acemoglu and Newman (2002) endogenize monitoring, showing that higher wages are associated with lower monitoring.

[8] In Shapiro-Stiglitz, it is the No Shirking Constraint (NSC) rather than the participation constraint that prevents the firm from punishing workers by lowering wages. Firms knew they could lower the wage, but they also knew if they lowered the wage below the NSC, the worker would shirk, so it never *paid* to lower wages. In multi-period principal agent models, terminations can be a more effective tool than wage cuts. See Stiglitz and Weiss (1983).

[9] Some of the earlier principal-agent literature did recognize that there were limits on the extent of punishment that could be imposed, and that this imposed constraints on the set of admissible contracts. See, for instance, the earlier sharecropping literature (Stiglitz (1974a), Newbery and Stiglitz (1979), and Braverman and Stiglitz (1982)), the early literature on labor compensation (Stiglitz, 1975), and that on financial contracts (Innes, 1990).

unemployment. The existence of such costs borne by the worker means that one can provide the requisite incentives with a lower level of unemployment than would otherwise be the case. But costs borne by the firm lower the demand for labor at any given turnover rate, and thus lead to higher unemployment.

It also makes a difference whether the consequences of the failure to exert effort are felt only in the period at hand. Low effort (e.g., in maintaining health) today could lower productivity for an extended period of time, in which case the observation of an adverse signal would suggest that the workers' productivity going forward would be lower. But then, if the adverse state were largely a result of exogenous shocks—not the individual's efforts—and if mobility costs were high, optimal contracting would entail some insurance, i.e., paying wages in excess of the individual's productivity in the event of a bad signal. In a competitive market, average pay has to equal average productivity, implying that when the adverse signal is not observed, the worker receives less than his productivity. Still, so long as the likelihood of being in the adverse state is affected by effort, the firm will want to engage in some punishment when the adverse signal is observed.[10] In contrast to Shapiro-Stiglitz, where the consequences of not exerting effort are felt only in the period in which effort is not exerted, the fact that the workers' expected productivity is low after the observation of an adverse signal makes firm punishment credible.

We show that if exogenous frictions are small, then there must be some unemployment in equilibrium, and this is even true when workers can be (and are) "punished" for poor outcomes, if even just by the lowering of wages. At the same time, so long as there are some frictions, the firm does not have to rely only on unemployment to discipline workers. It can create rents internally, with workers being punished by low wages. We describe the key determinants of equilibrium unemployment and wages, showing that up to a point, increasing frictions imposed on workers (putting "sand in the wheels") increases output and employment, and with appropriate redistribution policies, workers' well-being. When actual frictions are too small, it may be desirable

[10] In this sense, the paper is also a contribution to the theory of implicit contracts. See, e.g., Azariades and Stiglitz (1983), Newbery and Stiglitz (1987), and Arnott, Hosios and Stiglitz (1988).

to artificially create frictions.

The structure of the usual principal agent problem, where the firm maximizes its profits subject to the constraint that the contract generates expected utility to the worker at least equal to the workers' reservation utility level, gives a prima facie semblance of efficiency: profits cannot be increased further given workers' utility. But, as the previous paragraphs have emphasized, there is no presumption that the symmetric Nash equilibrium (where each firm takes the actions of others, and thus say the reservation wages and participation constraints as given) is efficient; to the contrary, in general, it will not be. Market equilibria with asymmetric information and imperfect risk markets are generically inefficient and this analysis of a principal agent model embedded in a general equilibrium model provides another concrete manifestation, one which suggests welfare enhancing policy interventions.[11]

[11] The intuition is simple: firms take, for instance, participation constraints (and here, constraints on punishment) as given, but they are in fact endogenous, i.e., affected by the behavior of *other* firms. This gives rise to a kind of externality which Greenwald and Stiglitz (1986,1988), Arnott and Stiglitz (1985), and Arnott, Greenwald and Stiglitz (1994) have shown matters.

II. The model

In this section we formulate a simple continuous-time model that captures the incentive role of unemployment as described above. Extensions and alternative assumptions are considered in subsequent sections.

1. Description

a. Workers

There is a mass N of infinitesimal, identical, workers, who dislike putting forth effort but enjoy consuming goods. Their utility is assumed to be time-separable, and their instantaneous utility function is given by $U(w, e)$, where w is the wage received and e is the level of effort on the job. For simplicity, we assume that workers are risk-neutral and that U is separable in w and e; with suitable normalization, we can therefore write workers' instantaneous utility as $U(w, e) = w - e$.[12] Workers are infinitely lived and maximize their expected present discounted value of utility,

$$W = E\left[\int_{\theta}^{\infty} U\big(w(t), e(t)\big) \exp(-rt)\, dt\right],$$

where $r > 0$ is the discount factor.[13] Again, for simplicity, we also assume that workers can either perform at a customary level of effort, $e > 0$, or shirk and provide minimal effort, $e = 0$.[14] When a worker is employed, there is an exogenous probability b per unit time that he enters the unemployment pool due to relocation, etc. He can also voluntarily quit his job or be fired (both are endogenous decisions which are discussed below). When a worker is unemployed, he receives unemployment benefits[15] of w_u (and $e = 0$) and there is a probability a per unit time that he gets a

[12] Under this assumption, allowing workers to borrow (or save) does not affect the analysis. Introducing risk-aversion still would not affect the analysis if workers cannot borrow nor save, or if they have constant absolute risk-aversion (CARA) utilities. Otherwise, risk-aversion would bring in wealth effects and thus heterogeneity among workers (which is discussed below), but also adverse selection problems if savings cannot be monitored (see Arnott-Stiglitz (1985) for an analysis of the impact of borrowing/saving and wealth effects on firms' turnover policies, and Chiappori *et al.* (1992) for an analysis of the impact of non-monitorable savings in repeated moral hazard situations).

[13] Introducing an exponential death rate would not alter the structure of the model.

[14] Introducing effort as a continuous variable would not change the qualitative results.

[15] We assume that these payments do not depend on whether the individual is fired or quit. In practice, many unemployment insurance schemes do make such distinctions.

job; the job acquisition rate a is an endogenous variable.[16] We also assume that there are exogenous switching costs: a worker bears a cost σ_w when leaving a firm (whether he is fired or quits) and a cost $\eta_w \geq 0$ when hired;[17] firms similarly bear costs of $\sigma_f \geq 0$ and $\eta_f \geq 0$.[18]

b. Firms

There are M identical firms. Within a firm, an operating unit is attributed to each worker. This unit can be either "effective" or not. Whether an operating unit is effective is observable. We assume for simplicity that an ineffective unit produces zero output and that the total output of the firm is a function of the number m of manned effective units, given by $f(m)$, where $f(\cdot) \geq 0$, $f''(\cdot) \leq 0$, $\lim_{m \to 0} f(\cdot) = 0$, $\lim_{m \to 0} f'(\cdot) = \infty$, and $\lim_{m \to \infty} f'(\cdot) = 0$.[19] Thus, if the firm has ℓ workers, it also has ℓ units, and if a proportion p of these units are effective, then its production is $f(p\ell)$. Besides the labor cost, each production unit (whether effective or ineffective) has carrying capital costs of $\rho \geq 0$. For simplicity, we assume all firms are identical. Hence, if there are L (employed) workers in equilibrium, each firm has $\ell = \frac{L}{M}$ workers,[20] and $m = \frac{pL}{M}$. The global production function, as a function of L,[21] is then $F(L) = Mf\left(\frac{pL}{M}\right)$ and satisfies $F'(L) = pf'\left(\frac{pL}{M}\right)$.[22]

If a worker's operating unit is currently effective, there is a probability per unit time that it

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[16] We assume that a does not depend on, say, the length of time that the individual is in the unemployment pool. Individuals are hired randomly from the unemployment pool. For alternative formulations, see Stiglitz (1969b).

[17] Alternatively, workers could leave at no cost and waive the unemployment benefits; we assume that joining the unemployment pool with benefits is a more attractive option – a sufficient condition is $w_u \geq r\sigma_w$.

[18] We can think of these as real costs, initially imposed on workers; they can however be shifted through contracting.

[19] One could alternatively interpret a worker's operating unit as the provision of some intermediate output at the individual level, in which case $f(\ell)$ describes the aggregation process linking the firm's output to individuals' intermediate outputs. Assuming that total production is also a function of the number of non-effective units would not alter the analysis, since in a long-run steady-state equilibrium, the numbers of effective and ineffective units are constant fractions (p and $1-p$, respectively) of the total number of units.

[20] Given our assumptions, the most efficient way to use a mass L of non-shirking workers is to allocate them evenly to the M firms.

[21] Later we will show that in equilibrium there is in general unemployment, so that L, the level of aggregate employment, is in general less than N, the labor supply.

[22] In the long run, the dynamic evolution becomes stationary and a given operating unit is effective with probability p_e. In the short-run, newly created units are initially ineffective. Though we focus here on steady states where firms are not creating new units, the assumption that newly created units are initially ineffective rules out artificial efficiency-enhancing effects.

becomes ineffective, equal to $q_e > 0$ if the worker exerts effort, and to $q_s > q_e$ if he does not. Conversely, if the unit is currently ineffective, there is a probability per unit time that it becomes effective, equal to p_s if the worker shirks, and to $p_e > p_s$ if he exerts effort. That is, the probability of an effective unit becoming ineffective is higher if the worker shirks and, conversely, the probability of an ineffective unit becoming effective is lower if the worker shirks.[23] Finally, unmanned units remain in the same state.[24]

The effectiveness of the worker's unit thus provides a noisy signal of his effort.[25] Even though the firm cannot observe whether the worker has shirked (otherwise wages would be contingent on effort) and knows that a low output (an ineffective production unit) may be the result not of shirking but of bad luck, it can nevertheless incentivize the worker to exert effort by making the wage contingent on the effectiveness of the worker's unit.

We assume that p_s is so low (and/or q_s is so high) that firms find it desirable to give workers incentives to provide effort. Hence, in a steady state equilibrium:

$$pq_e = (1-p)p_e.$$

For simplicity, we assume that $p_i = 1 - q_i$ for $i = e, s$, which implies:

$$p = p_e.$$

The assumption $p_i = 1 - q_i$ (for $i = e, s$) also means that the same wage schedule can induce effort regardless of the state of the unit (i.e., a worker in an ineffective unit is rewarded for making his unit productive in exactly the same way that a worker in an effective unit is punished for making his unit unproductive).[26]

In the steady state on which we focus below, with workers randomly leaving the firm

[23] Shirking thus only affects the future state of the production unit; in practice, it could also affect current output.
[24] Later, we will consider the possibility of turning the unit over for repair.
[25] The modelling challenge is to build a framework in which there is a contractible variable on which wages can be made contingent and which is related to workers' efforts, but only partially so. Similar insights would obtain if such a variable were observable but not verifiable (e.g., shirking may be observable by peers, but not by courts).
[26] What matters for this stationarity of the incentive problem is $q_s - q_e = p_e - p_s$, which amounts to $q_s + p_s = p_e + q_e$. The additional assumption $p_e + q_e = 1$ simplifies computations. For example, the proportion of effective units in the long run would otherwise be given by $\frac{p_e}{p_e + q_e}$.

because of some exogenous reasons, there is a corresponding proportional distribution of vacancies; when hired, workers are assumed to be randomly assigned to a unit with a vacancy.[27]

c. Equilibrium

We look for an equilibrium in which firms compete in output-contingent wages. We further focus on Markov perfect equilibria, in which firms' strategies only depend on the current state (namely, on the current effectiveness of the production unit to which a worker is assigned). More precisely, a firm commits itself to pay a wage w_h (where h stands for high) so long as the worker's unit is effective and to w_l (where l stands for low) otherwise. We also make the important assumption that the worker can <u>at any time</u> walk away from the firm and join the unemployment pool (that is, the worker cannot commit himself to stay in the firm). Mobility costs serve as a partial substitute for such a commitment device.

Equilibria can take one of two forms: either firms limit the extent of punishment so that workers don't walk (w_l is sufficiently high) or they do not. We refer to the former as the "internal incentive" equilibrium and to the latter as the firing equilibrium (since punishing workers so much that they quit is equivalent to being fired). The next two sections focus on the former case, while section 4 shows how the results are adapted for the firing equilibrium.

2. Equilibrium analysis

a. Hiring decisions

In the steady state equilibrium to be described more fully below, whenever a vacancy exists, the firm fills it. We need to study when it is profitable for the firm to do so. We define P_h and P_l as the discounted present value of the marginal profitability generated by hiring a worker onto an effective and an ineffective unit respectively. The marginal productivity of a worker is $f'(p_e\ell)$ when the unit is effective, and 0 otherwise. Thus, the firm initially loses money on the ineffective unit (so

[27] Without this assumption, firms would encounter a problem in hiring and keeping workers in ineffective units. There are other ways of resolving this conundrum, e.g., paying new workers assigned to the ineffective units a hiring bonus. See footnote 32 below.

long as it pays a newly hired worker a positive wage), but it pays to do that because if it appropriately incentivizes the worker, the unit will switch to becoming effective. We thus have:

$$P_l = -[\rho + w_l + b(\sigma_f + \eta_f)]t + \exp(-rt)[p_e t P_h + (1 - p_e t) P_l],$$
$$P_h = [f'(p_e l) - \rho - w_h - b(\sigma_f + \eta_f)]t + \exp(-rt)[q_e t P_l + (1 - q_e t) P_h].$$

Using $\exp(-rt) \simeq 1 - rt$ and solving for P_h and P_l yields:

$$rP_l = \frac{p_e[f'(p_e l) - w_h] - (q_e + r)w_l}{1 + r} - \rho - b(\sigma_f + \eta_f),$$

$$rP_h = \frac{(p_e + r)[f'(p_e l) - w_h] - q_e w_l}{1 + r} - \rho - b(\sigma_f + \eta_f).$$

A firm's labor demand is thus such that the profitability of an ineffective unit is just equal to the hiring cost, i.e., $P_l = \eta_f$ (recall that new units are initially ineffective – see footnote 22), or:[28]

$$p_e f'(p_e \ell) = (1 + r)[\widehat{w}_e + \rho + b(\sigma_f + \eta_f) + r\eta_f], \tag{1}$$

where

$$\widehat{w}_e \equiv \frac{p_e w_h + (q_e + r)w_l}{1 + r}$$

denotes the relevant expected wage.[29]

The interpretation of condition (1) is most straightforward in the case where r is small (we take the limit as r goes to zero): then hiring a worker in an ineffective unit gives the option of having an effective unit in the future, and indeed the unit will spend a fraction p_e of its future being effective and generating a net profit of $f'(\cdot) - w_h$, and the rest of the time losing w_l. In addition, the firm bears capital and mobility costs (per unit time) equal to $\rho + b(\sigma_f + \eta_f)$. Condition (1) simply says that the losses incurred when the unit is ineffective are just offset by the profits when it is effective, taking into

[28] The marginal profitability of an effective unit is then $P_h^* = \frac{w_l^* + \rho + b\sigma_f + (b + r + p_e)\eta_f}{p_e} > 0.$

[29] The expected discounted wage bill is indeed equal to $\frac{\widehat{w}_e}{r}$.

account mobility costs and making appropriate adjustments for "delay": as the ineffective unit does not instantaneously convert into an effective one, interest must be paid on the implied investment.

The aggregate demand for labor $L = D(\widehat{w}_e)$ satisfies $F'(L) = p_e f'\left(p_e \frac{L}{M}\right) = p_e f'(p_e \ell)$, which, using (1) and $v \equiv \frac{L}{N-L}$, amounts to:

$$F'\left(\frac{Nv}{1+v}\right) = (1+r)\left[\widehat{w}_e + \rho + b\left(\sigma_f + \eta_f\right) + r\eta_f\right]. \qquad (D)$$

Condition (D) can be thought of as the aggregate "labor demand", giving for each value of the "average wage" $\widehat{w}_e$, the corresponding labor demand by firms, captured here by the variable v.

Finally, letting

$$w_e \equiv p_e w_h + q_e w_l = w_l + p_e(w_h - w_l)$$

denote the average wage paid to employed workers, the industry profit, Π, is such that:

$$r\Pi = F\left(D(\widehat{w}_e)\right) - \left[w_e + \rho + b\left(\sigma_f + \eta_f\right)\right]D(\widehat{w}_e).$$

b. Worker value functions

Let V_j denote the discounted sum of expected utility for a worker who always provides effort, according to whether his unit is currently effective $(j = h)$ or not $(j = l)$. These values can be expressed as functions of V_u, the expected utility of a currently non-employed worker (which in steady state equilibrium will be stationary).[30] Considering first a worker whose unit is currently effective and looking at a short time interval $[0, t]$, we have:

$$V_h = (w_h - e)t + \exp(-rt)\{bt(V_u - \sigma_w) + (1 - bt)[q_e t V_l + (1 - q_e t)V_h]\},$$

since the worker gets a wage w_h but faces a probability bt of leaving the job during the interval $[0, t]$ and, if not, faces during this same interval a probability $q_e t$ that his unit becomes ineffective. Solving for V_h, we have:

[30] These expected utilities are "gross utilities," before taxes. We assume throughout the analysis that taxes (such as those needed to finance the unemployment benefit scheme) are lump-sum ones and do not interfere with the incentives problem under scrutiny.

$$V_h = \frac{(w_h - e)t + \exp(-rt)[bt(V_u - \sigma_w) + (1 - bt)q_e t V_l]}{1 - \exp(-rt)(1 - bt)(1 - q_e t)}.$$

Using again $\exp(-rt) \simeq 1 - rt$:[31]

$$rV_h = w_h - e + b(V_u - \sigma_w - V_h) + q_e(V_l - V_h). \quad (2)$$

Similarly, we have:

$$rV_l = w_l - e + b(V_u - \sigma_w - V_l) + p_e(V_h - V_l). \quad (3)$$

Lastly, V_u is characterized by:[32]

$$rV_u = w_u + a(V_l - \eta_w - V_u) + ap_e(V_h - V_l),$$

noting that unemployed workers obtain a job with probability a per unit time, and that a proportion p_e of available positions are attached to effective units.

In a steady state in equilibrium, the equality between unemployment flows in and out gives $a(N - L) = bL$ or in terms of our earlier notation:

$$a = bv,$$

enabling us to rewrite the above expression of rV_u as

$$rV_u = w_u + bv(V_l - \eta_w - V_u) + bvp_e(V_h - V_l) = w_u + bv(V_e - \eta_w - V_u), \quad (4)$$

where

$$V_e \equiv (1 - p_e)V_l + p_e V_h = V_l + p_e(V_h - V_l)$$

denotes the average discounted utility of employed workers. Solving this equation for V_u yields:

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[31] This equation is a standard "asset equation", of the form "interest rate times asset value equals flow benefits plus expected capital gains".

[32] We assume that when they are hired, workers are randomly assigned to available positions; hence, in equilibrium, a fraction p_e are assigned to an effective unit. While this assumption seems reasonable if there are high switching costs when "moving" workers inside a firm, different hiring strategies would affect unemployed workers' perspectives and would thus alter the equilibrium; for example, assigning newly hired workers to ineffective units and using available effective units to "promote" inside workers would increase inside turnover costs but ease the participation constraint described below. Introducing limited internal switching costs would thus open a new dimension and in particular allow the analysis of career management strategies.

$$V_u = \frac{w_u + bv(V_e - \eta_w)}{r + bv}.$$

In particular, $V_u = \frac{w_u}{r}$ when there is no employment (i.e., $v = 0$, implying that workers remain unemployed forever) and $V_u \simeq V_e - \eta_w$ when there is full employment (i.e., $v \longrightarrow \infty$, implying that currently unemployed workers can immediately find a job).

c. Relevant constraints

Three relevant constraints determine the wages that can be offered: (i) employed workers must be incentivized to exert effort (no shirking), and (ii) to stick to their firm when their units become ineffective (no bondage), and (iii) unemployed workers must be willing to accept job offers (employment constraint).

• *No-shirking constraint (NS)*. Workers provide effort only if they are given incentives to do so. Consider for example a worker whose unit is currently effective. By shirking during a short time interval $[0, t]$, he can expect to get:

$$w_h t + \exp(-rt)\{btV_u + (1 - bt)[q_s tV_l + (1 - q_s t)V_h]\}.$$

If instead he decides not to shirk, he expects to get:

$$V_h = (w_h - e)t + \exp(-rt)\{btV_u + (1 - bt)[q_e tV_l + (1 - q_e t)V_h]\}.$$

Using $p_i = 1 - q_i$ for $i = e, s$, the no-shirking constraint can thus be written as:

$$et \leq \exp(-rt)(1 - bt)(p_e - p_s)t(V_h - V_l),$$

which, using $\exp(-rt) \simeq 1 - rt$, amounts to:

$$(p_e - p_s)(V_h - V_l) \geq e. \tag{NS}$$

That is, the expected gain from exerting effort must exceed its cost. This requires a large enough difference in the wages attached to effective and ineffective operating units: condition (NS) implies $V_h > V_l$, which in turn requires $w_h > w_l$.[33] Incentives require the employer to "reward" the worker when his unit is effective compared to when it is not.

[33] Using (2) and (3), (NS) can be written as $(p_e - p_s)(w_h - w_l) \geq (b + r + 1)e$.

A similar reasoning for a worker whose unit is currently ineffective yields the same condition.[34] Condition (NS) thus constitutes the only relevant no-shirking condition. Heuristically, (NS) implies that for any w_l, there is a minimum w_h which induces effort: $w_h \geq \Phi(w_l)(> w_l)$. In our model, what induces effort is not the level of wages, but the disparity between wages associated with good and bad performance.

• *No-bondage constraint (NB)*. So far, the analysis is similar to earlier analyses of shirking models. What the earlier literature failed to emphasize sufficiently was the importance of the *no-bondage* constraint—the fact that workers can always leave a firm. Since workers can always walk away from the firm (at cost σ_w), there is a limit to the penalty that can be imposed for shirking if the worker who is found shirking (or for whom an adverse signal has been observed) is not to leave.

More formally, the no-bondage constraint for a worker in an ineffective work unit can be written as:[35]

$$V_l \geq V_u - \sigma_w. \qquad \text{(NB)}$$

Combining (NS) and (NB) defines the minimal expected wage that must be offered to prevent shirking *and* to retain the worker. That is, given σ_w (the cost of leaving), (NB) implies that w_l must be greater than a certain level, which depends on the utility that individuals get if they enter the unemployment pool. This utility itself depend on the wages offered by other firms. In reduced form, we can write:

$$w_l \geq \psi(\sigma_w; w_h^*, w_l^*, p^*, u),$$

where w_i^* represents the wages paid by other firms, p^* represents the fraction of effective units in the representative firm, and u is the unemployment rate. Clearly, $\psi_{\sigma_w} < 0$: higher exit costs imply that the firm can impose stronger punishment by paying a lower wage to workers in ineffective work units.

[34] A stationarity argument shows that when condition (NS) holds, no other shirking strategy (e.g., shirking for a longer period) yields positive gains. In particular, (NS) implies $V_j \geq V_j^s$ for $j = h, l$, where V_j^s denotes the expected utility of an all-time shirker whose unit is currently effective ($j = h$) or not ($j = l$).

[35] The no bondage constraint written here corresponds to an *ex post* participation constraint, from the viewpoint of an already employed worker—knowing whether his production unit is effective. The *ex ante* participation constraint (from the point of view of an unemployed worker, when offered a job – in effect, an *employment constraint*) is discussed below. Also, recall that joining the unemployment pool is supposed to be more attractive than just walking away from the firm – see footnote 17; thus, the no-bondage condition also implies $V_l \geq 0$.

For a sufficiently large value of σ_w, the worker will not quit. Moreover, $\psi_u < 0$: the higher the unemployment rate, the lower the punishment wage the firm can pay without the worker quitting, given what other firms are paying.

• *Employment constraint (EC)*. For a given w_l and w_h, we can calculate the average V_e, the expected utility of a newly hired worker. Recall from our earlier discussion that we assume workers are randomly assigned to vacancies.[36] A worker will accept an offer only if it pays him to leave the unemployment pool, i.e.:

$$V_e - \eta_w \geq V_u. \tag{EC}$$

If there exists a steady-state equilibrium contract of the form posited, it has to satisfy all the above conditions: in essence, the lower wage w_l must be high enough to satisfy the no bondage constraint, the bonus $w_h - w_l$ high enough to elicit effort, and the average compensation w_e high enough to induce people to leave unemployment.

Intuitively, if it is not too costly to take a job, then if workers assigned to ineffective units (and thus paid the lower wage) are willing to stay, unemployed workers should a fortiori be willing to accept the offered positions. For all of these conditions to be satisfied simultaneously requires some restrictions on the parameters of the problem. The precise condition is captured by the following assumption:

Assumption A:

$$\eta_w + \sigma_w < B,$$

where

$$B \equiv \frac{p_e e}{p_e - p_s}$$

denotes the expected bonus required to incentivize workers to exert effort.

Our first insight is that we can ignore the employment constraint, which in turn implies that

[36] In steady state, vacancies only arise because of exogenous quitting, and thus reflect the relative proportion of effective and ineffective units.

the no-shirking condition is binding and that there is unemployment in equilibrium:

Proposition 1: Under Assumption A:

(i) (EC) can be ignored, implying that (NS) and (NB) are both binding.

(ii) There is unemployment in the no-firing equilibrium.

The proof is straightforward. For (i), it suffices to note that, under Assumption A, (NS) and (NB) together imply (EC), which can therefore be ignored. It follows that (NS) must be binding, as this is the only constraint that may prevent firms from reducing w_h, and (NB) must be binding as well, as this is the only constraint that may prevent firms from reducing w_l.[37]

Turning to (ii), first note that (NB) and (NS) together imply:

$$V_e \geq V_u - \sigma_w + B.$$

In the case of full employment, unemployed workers could immediately find a job, and so $V_u \simeq V_e - \eta_w$. We would thus have $V_e \geq V_e - \eta_w - \sigma_w + B$, contradicting Assumption A.

It follows from Proposition 1 that, if exogenous friction costs are not too high (namely, Assumption A holds), then the provision of sufficient incentives in the no-firing equilibrium requires endogenous friction in the form of unemployment.

III. The internal incentive equilibrium (no-firing)

1. Equilibrium outcome

[37] The analysis can be adapted in a straightforward way to incorporate a minimum wage: introducing a binding minimum wage would replace the no bondage constraint and raise further the "punishment" wage w_l. Together with the no shirking condition, this would raise w_l and the expected wage as well, thus reducing employment. Relaxing the no bondage constraint would instead reduce average wages and boost employment. Conversely, the no bondage constraint could be partly relaxed to allow firms to impose (endogenous) leaving fees. Firms would then have an incentive to introduce such fees (compensating upfront the hired workers), as this would enable them to lower the punishment wage. Doing so would also reduce average wages (even accounting for the upfront compensation) and boost employment.

a. Characterization

We can therefore consider (NS) and (NB) as binding constraints. Straightforward substitution yields:

$$rV_u = w_u + \beta v, \tag{5}$$

where

$$\beta \equiv b[B - (\sigma_w + \eta_w)] > 0,$$

with the inequality following from Assumption A. Further manipulation of the relevant inequalities yields (recalling the earlier definition of $\widehat{w}_e$) a simple linear relationship between $\widehat{w}_e$ and v:

$$\widehat{w}_e = \alpha + \beta v, \tag{S}$$

where

$$\alpha \equiv w_u + e + \frac{bB}{1+r} - r\sigma_w.$$

Condition (S) can be thought of as the "labor supply" giving, for each value of v, the necessary value of the "average wage" needed to sustain both effort and workers at ineffective units not quitting.

We have now completely described the no-firing equilibrium (if it exists): it is the simultaneous solution to this supply equation and the earlier defined demand equation, which for simplicity we repeat here:

$$F'\left(\frac{Nv}{1+v}\right) = (1+r)\left[\widehat{w}_e + \rho + b\left(\sigma_f + \eta_f\right) + r\eta_f\right] \tag{D}$$

It readily follows that the equilibrium is "interior": as v tends to zero, the left-hand side tends to infinity, whereas the right-hand side remains finite; conversely, as already noted by Proposition 1, v cannot tend to infinity – indeed, the left-hand side would then tend to zero, whereas $\widehat{w}_e$ and the right-hand side would both tend to infinity.

What is striking about the equilibrium is that the effects of the parameter changes on the equilibrium output and employment levels through the supply side are all mediated through their impacts on two variables reflecting the level and slope of the modified supply curve, α and β. Some of the parameters affect only α and some only β. In those cases, comparative statics (to which we come shortly) are straightforward. In some cases, changes in parameters affect both α and β; then the analysis becomes more complicated.

Figures 1 and 2 show the supply and demand curves, (*S*) and (*D*), in the $(L, \widehat{w}_e)$ and $(v, \widehat{w}_e)$ spaces, respectively. Their intersection determines the market equilibrium, $(L^*, \widehat{w}_e^*)$ or $(v^*, \widehat{w}_e^*)$.

Having solved for the equilibrium value of v^* (and thus u^*), we can then use our earlier results to explicitly derive the equilibrium wages, the utility levels of unemployed and employed workers, and profits. We can then ascertain the effects of changes in any of the parameters on the equilibrium values of profit and expected utility.

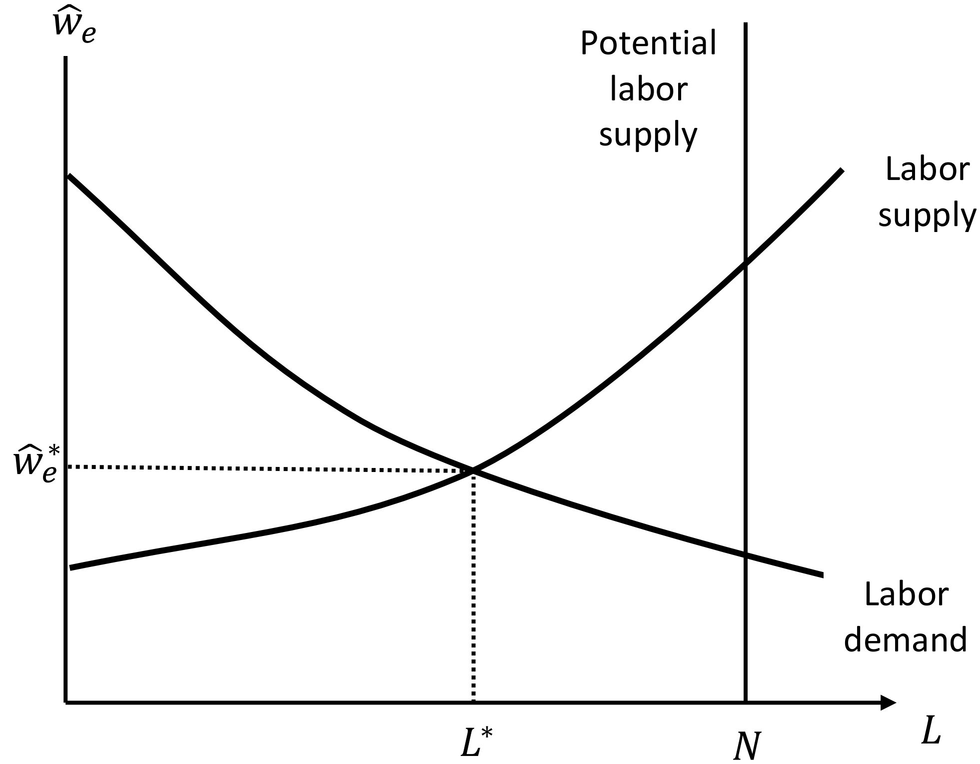

Figure 1: Market equilibrium in the $(L, \widehat{w}_e)$ space. The labor demand (*D*) is similar to the standard demand curve. By contrast, while the potential supply of labor is fixed and equal to N, implying that the standard supply curve would be perfectly inelastic, the actual supply of labor embeds the incentive constraints (no shirking and no bondage), which make it more elastic than the standard supply curve. The market equilibrium, $(L^*, \widehat{w}_e^*)$, entails unemployment ($L^* < N$).

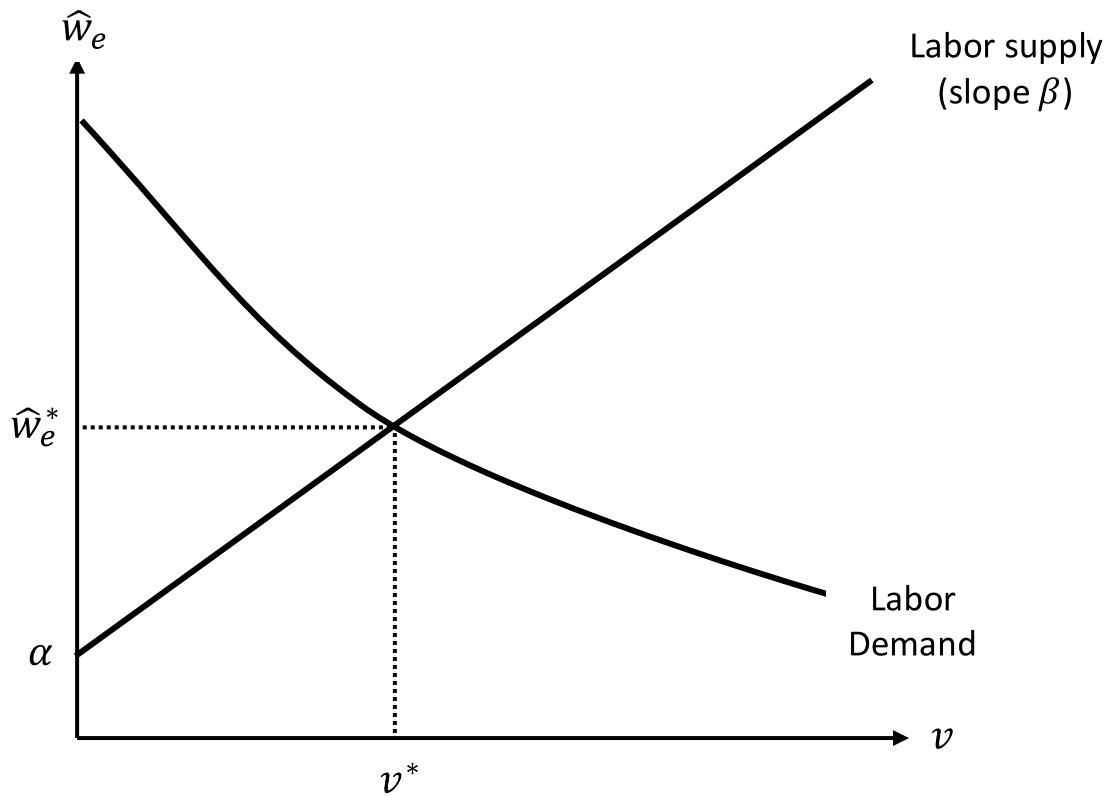

Figure 2: Market equilibrium in the $(\boldsymbol{v}, \widehat{\boldsymbol{w}}_{\boldsymbol{e}})$ space. The modified supply curve is linear in this space. The demand curve remains downward sloping.

b. Comparative statics

A change in any parameter affecting α or β thus affects the equilibrium wage and employment as follows:

$$d\widehat{w}_e^* = \frac{d\alpha + \ v^* d\beta - \beta\lambda^*\varepsilon^*\left[\left(\sigma_f + \eta_f\right)db \ + bd\left(\sigma_f + \eta_f\right) \ + rd\eta_f \right]}{1 + \beta\lambda^*\varepsilon^*},$$

$$dv^* = -\frac{\lambda^*\varepsilon^*}{1 + \beta\lambda^*\varepsilon^*}\left[\ d\alpha + v^* d\beta + \left(\sigma_f + \eta_f\right)db \ + bd\left(\sigma_f + \eta_f\right) \ + rd\eta_f \ \right].$$

where $\varepsilon^* \equiv \frac{\widehat{w}_e^* D'(\widehat{w}_e^*)}{D(\widehat{w}_e^*)} = \frac{(1+r)\widehat{w}_e^*}{F''(L^*)L^*}$ represents the elasticity of the demand for labor, evaluated at equilibrium, and $\lambda^* \equiv \frac{(1+v^*)v^*}{\hat{w}_e^*}$. A decrease in either the intercept (α) or the slope (β) of the supply curve therefore leads to a decrease in the equilibrium wage, an increase in profits, a reduction of the level of unemployment, and thus to an increase in total output, as illustrated in Figure 3. Other things being equal, the effect on unemployment is larger and the impact on wages is lower when the demand for labor is more elastic.

We can analyze the determinants of α and β.

Agency cost. The variables α and β are both increasing in the required incentive bonus B: raising B makes it more difficult to incentivize effort, which raises equilibrium wages and reduces equilibrium employment.

Friction costs. Raising the friction costs faced by *workers* reduces the need for the endogenous friction of unemployment needed to ensure that a low wage is acceptable. Specifically, a higher exogenous friction enables a lower "punishment" wage and, thus, a lower average wage (maintaining the wage differential to provide the requisite bonus), which in turn reduces unemployment.

By contrast, raising the friction costs faced by *firms* has no impact on the labor supply (S) and depresses the labor demand (D), by increasing the cost of labor due to exogenous turnover; as a result, the equilibrium wages and equilibrium employment both decrease.

Turnover Rate. An increase in the rate b of exogenous turnover depresses both labor supply (by raising both α and β) and labor demand: it lowers the value of being employed, which exacerbates the agency problem, and also lowers profitability (by increasing the occurrence of the friction).

Unemployment benefit. Finally, increasing the unemployment benefit w_u raises the value of being unemployed, which again exacerbates the incentives problem, raises equilibrium wages, and reduces equilibrium employment.

These results (except the one on frictions) parallel those for the standard efficiency wage (Shapiro-Stiglitz) model.

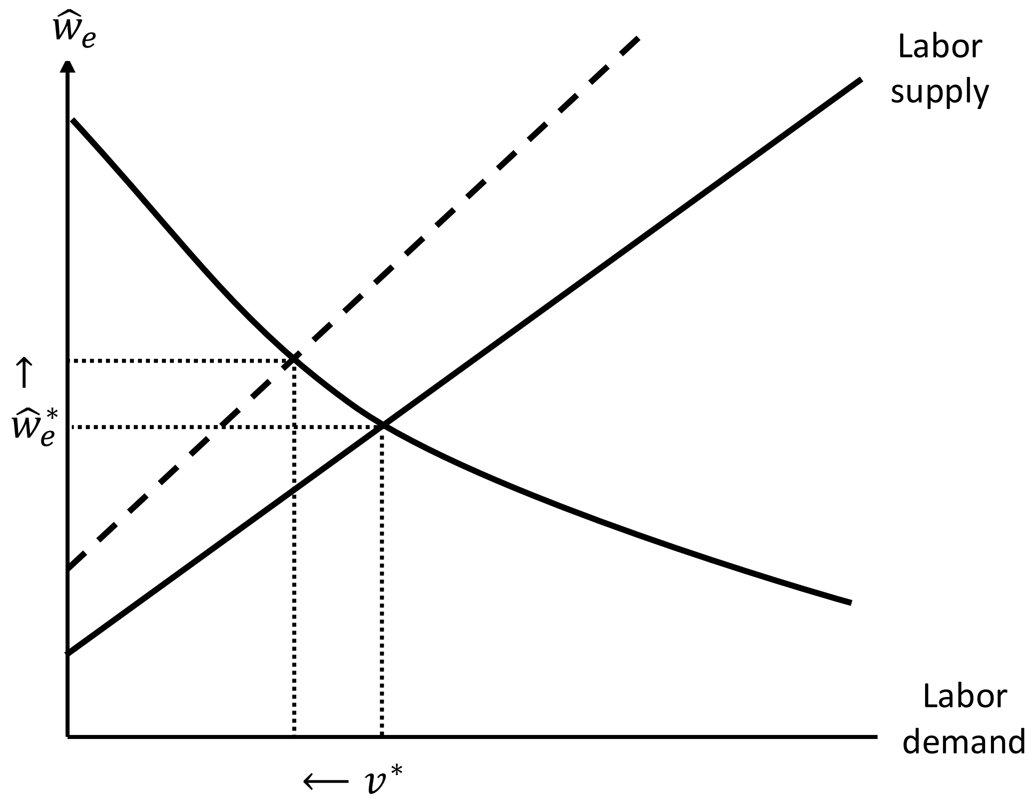

Figure 3. Comparative statics. A shift upwards of the modified supply curve or an increase in its slope leads to higher wages and unemployment.

2. Welfare analysis

Parameter changes leading to lower wages are (other things constant) bad for workers and good for profits; and parameter changes leading to lower unemployment (again other things equal) would be expected to be good for welfare. But other things are never equal: increasing costs of frictions lead to lower unemployment, but when workers experience turnover, their costs are higher. Hence, one needs a fuller analysis. The calculations are straightforward and presented in Appendix A. Importantly, we can identify both *direct* effects of parameter changes and the indirect (general equilibrium) effects, through changes in the unemployment rate and wages.

a. Workers' utilities

The easiest expression to derive is that for the change in V_u^*, where the only indirect effect is through the change in v^*:

$$rdV_u^* = \frac{dw_u + v^* d\beta - \beta\lambda^*\varepsilon^*\left[d\alpha + \left(\sigma_f + \eta_f\right)db + bd\left(\sigma_f + \eta_f\right) + rd\eta_f\right]}{1 + \beta\lambda^*\varepsilon^*}.$$

From (5), a reduction in α increases unemployed workers' discounted utility *because it lowers the unemployment rate*, and this effect is larger when labor demand is more elastic. By contrast, a reduction in either the unemployment benefits w_u or the slope β of the supply curve reduces unemployed workers' utility *even though it increases the level of employment*; the effect is however lower when the demand for labor is more elastic. The reduction in β is associated with an increase in workers' mobility costs, which hurts workers.

Recalling the definition of the expected discounted utility of employed workers, V_e, and using again the binding (NB) and (NS), it is straightforward to show that:

$$dV_e^* = dV_u^* + dB - d\sigma_w.$$

The impact on employed workers' utility is therefore similar to that on the utility of unemployed ones, except that in the former there is an additional effect from alterations of B (a higher B forcing firms to pay a higher w_h) or σ_w (increasing worker friction allows firms to lower wages).

Finally, the equilibrium value of total discounted aggregate profit, Π^*, satisfies:

$$r\Pi^* = F\left(D(\widehat{w}_e^*)\right) - \left[w_e + \rho + b\left(\sigma_f + \eta_f\right)\right]D(\widehat{w}_e^*),$$

where, as before, $w_e \equiv p_e w_h + q_e w_l$ is the average wage paid to employed workers. Differentiating yields:

$$rd\Pi^* = -L^*\left[r\frac{w_l^* + \rho + b\left(\sigma_f + \eta_f\right) + (1+r)\eta_f}{\widehat{w}_e^*}\varepsilon^* d\widehat{w}_e^* + dw_e^* + \left(\sigma_f + \eta_f\right)db + bd\left(\sigma_f + \eta_f\right)\right],$$

The impact on profits is thus inversely correlated with that on wages. We can again study the impact of the main parameters.

Friction costs. As we have seen, an increase in firms' exogenous mobility costs reduces wages and employment, and thus harms workers. By contrast, the effects of an increase in workers' frictions would appear to be more ambiguous, with an adverse direct effect but a positive indirect effect. However, the direct effects of σ_w and η_w dominate.

Unemployment benefits. An increase in the unemployment benefits w_u shifts the supply curve upwards, leading to higher equilibrium wages and unemployment. Despite yielding a higher unemployment rate, an increase in w_u increases workers' expected utilities. *The direct impact of higher unemployment benefits thus more than offsets their undesirable effect on unemployment, and this is true even in our model, which does not fully incorporate workers' risk aversion*. Indeed, we have:

$$r\frac{\partial V_e^*}{\partial w_u} = r\frac{\partial V_u^*}{\partial w_u} = \frac{1}{1+\beta\lambda^*\varepsilon^*} > 0.$$

b. Total welfare

Total welfare, defined as the sum of workers' expected utility and profits, taking into account the costs of unemployment benefits, satisfies:

$$rW \equiv LrV_e + (N-L)rV_u + r\Pi - (N-L)w_u$$

$$= F(L) + L\big[rV_e - w_e - b\big(\sigma_f + \eta_f\big)\big] + (N-L)(rV_u - w_u),$$

Using (2), (3), (4) and $(N-L)v = L$, its equilibrium value is such that:

$$W^* = F(L^*) - L^*\big[e + \rho + b\big(\sigma_w + \eta_w + \sigma_f + \eta_f\big)\big].$$

Societal welfare is equal to output less (a) the cost of the effort required to produce the output; (b) capital costs for the machines) and (c) total labor turnover costs. Parameter changes again have direct and indirect (general equilibrium) effects, but now we take into account the fact that some of the indirect effects are distributive, with losses, say, to workers, offset by gains to employers. If we were to think of workers as simultaneously the owners of the firms, these distributive effects would net out. The following results are best interpreted in that way (details are again presented in Appendix A).

The indirect effect is simply:

$$\frac{\partial W^*}{\partial L^*} = \gamma^* \equiv F'(L^*) - e - b\left(\sigma_w + \eta_w + \sigma_f + \eta_f\right) > 0,$$

that is, it is measured by the equilibrium productivity of labor, net of the cost of effort and turnover, where the inequality follows from (D) and Assumption A. γ^* is the *equilibrium net marginal surplus of* *production*.

Firm frictions. An increase in firm frictions lowers W^* both directly and indirectly (through an increase in unemployment) and so is welfare decreasing.

Worker frictions. By contrast, an increase in worker frictions, by reducing the need for costly unemployment, may improve W^* if the *net* productivity γ^* or the elasticity of demand ε^* is high enough (so the induced employment effect is large enough). Specifically, an increase in η_w is welfare increasing if and only if the equilibrium net surplus is large enough:

$$\gamma^* > (1+v^*)\left(\beta + \frac{1}{\lambda^* \varepsilon^*}\right),$$

which, if $\gamma^* > (1+v^*)\beta$, will be true if the elasticity of demand is great enough:

$$\varepsilon^* > \frac{1+v^*}{\lambda^*[\gamma^* - (1+v^*)\beta]}.$$

We have seen that, compared with an increase in the hiring cost η_w, increasing the leaving cost σ_w has larger effects on unemployment[38] for the same overall effects on turnover costs: as a result, the overall effect of an increase in σ_w is more likely to be beneficial, with now the critical condition being:

$$\gamma^* > (1+v^*)\left(\beta + \frac{1}{\lambda^* \varepsilon^*}\right)\left(\frac{bv^*}{r+bv^*}\right),$$

where the right-hand side is lower than in the previous threshold.

It follows from this analysis imposing fees on workers either as they got hired or fired, with proceeds redistributed, could increase welfare. [We discuss this further below.]

Unemployment benefits. It also follows directly that increased unemployment benefits (which are just transfers from profits to workers) lowers W^*. We hasten to add, however, that this model does not incorporate the central reasons for having unemployment insurance, namely the risk aversion of workers and their inability to smooth income across states or dates. The analysis is thus only

[38] Recall that an increase in η_w only reduces the slope β of the supply curve, whereas σ_w also reduces the level α (and has the same impact on the slope β).

identifying the adverse agency costs.

3. Policy determined separation / mobility costs

In the analyses of the previous sections, we have seen how throwing sand in the wheels – at least the right kind of sand, namely, sand on the worker side – lowers unemployment and can improve social welfare. We have explored *real* costs of mobility. But public policy can also affect mobility, and the revenues raised from taxes on firing or hiring, borne either by workers or firms, can be used for public purpose. This changes the calculus dramatically. In particular, it is straightforward to show, say in the case of the no-firing equilibrium, that the resulting equilibrium value of welfare, $\widehat{W}^*$, is such that:

$$\frac{d\widehat{W}^*}{d\tau_i} = \hat{\gamma}^* \frac{dL^*}{d\tau_i},$$

where $\hat{\gamma}^* > \gamma^*$ (as mobility costs, being transfers, are now less socially costly), implying from our earlier discussion that $\hat{\gamma}^* > 0$. The effect of a tax τ_i on hiring or firing, imposed on workers or firms, on welfare depends simply on its impact on employment. Our previous analyses of comparative statics have provided clear answers: mobility costs imposed on firms reduce employment and those imposed on workers increase equilibrium employment.

IV. The external incentive equilibrium (firing workers)

The firm always has the option of firing a worker – or offering him a wage so low that he quits. In the previous section, we assumed that a "manned" ineffective unit is more likely to become effective if workers exert effort; we can think of the workers as exerting efforts at repair. But as an alternative, the firm can engage in direct repair expenditures, which can be either less costly or more effective. For simplicity, we assume that the maintenance operations have a success probability equal to that of a manned ineffective unit, and cost a fixed amount $\bar{w}$.[39] Of course, if the firm fires the worker or induces him to quit, it will incur a cost of σ_f and the worker may incur a cost as well. We first explore what a firing equilibrium looks like, if it exists. The following section

[39] These assumptions can be easily generalized. For example, a fuller model would allow the probability of transition from ineffective to effective for "repair" to differ from that for a non-shirker.

then asks: in the putative no-firing equilibrium, would a firm choose to fire, or conversely, in the putative firing equilibrium, would a firm choose not to fire?

1. Equilibrium analysis

a. Hiring decisions

The aggregate production function can now be written (using the same notation as before, but using the subscript to denote the "firing" equilibrium):

$$F_f(L) \equiv Mf\left(\frac{L}{M}\right).$$

However, there are now higher capital costs because of the inoperative units. Netting these and the maintenance costs out, we noting that if the firm has $\ell = \frac{L}{M}$ workers and, thus, as many productive units, it must have a total of $\frac{\ell}{p} = \frac{L}{pM}$ units. This is because at any moment in time, a fraction p of its total units are effective. The dynamic equations for profitability differ in five respects from those earlier:

(a) An ineffective machine now bears a repair cost $\overline{w}$ while it remains ineffective (instead of the punishment wage w_l).

(b) When the machine becomes ineffective, there is now a switching (firing) cost σ_f.

(c) When the machine becomes effective, there is a hiring cost η_f.

(d) Since there are no workers at ineffective units, ineffective units bear no mobility costs, whereas in the no-firing equilibrium, there is a flow of costs of $b(\sigma_f + \eta_f)$.

(e) Because there are more machines than workers (workers are only assigned to the effective machines), there are additional capital costs ($\frac{\ell}{p}\rho$ rather than $\ell\rho$).

Using the same methodology as before, we can solve for the expected profitability of effective and ineffective units, P_{hf} and P_{lf}, where "*f*" denotes the firing equilibrium.

$$P_{hf} = \frac{(p_e + r)\left[f'(\ell) - (w+\rho) - b(\sigma_f + \eta_f) - q_e\eta_f\right] - q_e(\overline{w} + \rho + p_e\eta_f)}{r(1+r)},$$

$$P_{lf} = \frac{p_e\left[f'(\ell) - (w+\rho) - b(\sigma_f + \eta_f) - q_e\eta_f\right] - (q_e + r)(\overline{w} + \rho + p_e\eta_f)}{r(1+r)}.$$

where w is the wage paid at all effective units in the firing equilibrium (there is now no wage paid at ineffective units.) As before, in equilibrium, it has to pay for firms to maintain an ineffective unit, so that $P_{lf} = 0$ or:[40]

$$f'(\ell) = w + \frac{q_e + r}{p_e}\overline{w} + \frac{1+r}{p_e}\rho + (b + q_e)\sigma_f + (b + q_e + r)\eta_f. \qquad (6)$$

The interpretation of (6) follows that of (1) for the case of no-firing. It is again seen most simply by taking the case of r being small (zero): when the firm calls in the maintenance operator, it incurs a loss of $\overline{w}$ until the unit becomes effective. Over its future life, the production unit will spend a fraction p_e of the time being productive, during which it will have a profitability, net of mobility costs of $f'(\ell) - w$. [Because of our simplifying assumption, the fraction of units that are effective is the same as before: $p = p_e$.] A fraction $1 - p_e$ of the time the unit has zero productivity, incurring a loss of $\overline{w}$ (rather than w_l). Mobility costs are higher now, as we have noted: per unit of time, they amount to $p_e(b + q_e)\left(\sigma_f + \eta_f\right)$. Again, there is an adjustment when $r > 0$, to reflect the fact that losses are upfront.

Using $F_f'(L) = f'\left(\frac{L}{M}\right) = f'(\ell)$ and $v = \frac{L}{N-L}$, (6) can be rewritten to parallel (D) for the no firing equilibrium:

$$p_e F_f'\left(\frac{Nv}{1+v}\right) = (1+r)\left(\widehat{w}_f + \rho\right) + p_e\left[(b + q_e)\sigma_f + (b + q_e + r)\eta_f\right],$$

where

$$\widehat{w}_f \equiv \frac{p_e w + (q_e + \mathrm{r})\overline{w}}{1+r}$$

denotes the "effective" wage. That is, the cost of the repair operator simply replaces the low wage worker, with an adjustment for the slightly higher mobility costs. The equilibrium employment equation is changed, as we suggested above, both as a result of the change in the "effective" wage and the change in mobility costs.

[40] The marginal profitability of an effective unit is then $P_{hf} = \frac{\overline{w}+\rho}{p_e} + \eta_f > 0$.

b. Worker value functions

There are two main differences compared with the no-firing case. First, workers are only assigned to effective units. Second, the labor turnover rate is higher, as employed workers join the unemployment pool whenever their unit becomes ineffective (on the other hand, there is no exogenous quitting from ineffective units anymore). The flow of labor into the unemployment pool is therefore faster, implying that the job acquisition rate is also faster, for any value of u (or v). Indeed, in steady state we now have $a(N-L)=(b+q_e)L$, or:

$$a=(b+q_e)v.$$

This, in turn, means that the unemployment rate required to sustain effort will have to be higher than in the firing equilibrium (and in the standard Shapiro-Stiglitz model).

Applying the same methodology as before shows that the expected utilities for employed and unemployed workers now satisfy:

$$rV_{ef}=w-e-(b+q_e)\left[V_{ef}-\left(V_{uf}-\sigma_w\right)\right], \tag{7}$$

$$rV_{uf}=w_u+(b+q_e)v\left(V_{ef}-\eta_w-V_{uf}\right), \tag{8}$$

where V_{ef} is the expected utility of an employed worker in the firing equilibrium.

c. Relevant constraints

Obviously, the non-bondage constraint is no longer relevant, as workers are fired or induced to quit. There are therefore only two potentially relevant constraints: the no-shirking constraint and the employment constraint.

• *No-shirking constraint (NS$_f$)*. The no shirking constraint is here different, because the penalty for not exerting effort is the increased probability of being ineffective and now facing unemployment, with the associated mobility costs:

$$(p_e-\ p_s)\left[V_{ef}-\left(V_{uf}-\sigma_w\right)\right]\geq\ e.$$

We can rewrite this constraint as:

$$V_{ef}-\left(V_{uf}-\ \sigma_w\right)\geq B_f, \tag{NS$_f$}$$

where the required bonus is now given by:

$$B_f \equiv \frac{e}{p_e - p_s}\left(= \frac{B}{p_e} > B\right).$$

• *Employment constraint (*EC_f*)*. As before, a worker will accept an offer only if it pays him to leave the unemployment pool, i.e.:

$$V_{ef} - \eta_w \geq V_{uf}. \qquad (\mathrm{EC}_f)$$

The same logic as before shows that, when exogenous friction costs as small, there must be unemployment (i.e., endogenous friction) in equilibrium:

Proposition 2: Under Assumption A:

(i) (EC_f) can be ignored, implying that (NS_f) is binding.

(ii) There is unemployment in the firing equilibrium.

The proof is again straightforward. For (i), it suffices to note that, under Assumption A, (NS_f) implies (EC_f), which can therefore be ignored. It follows that (NS_f) must be binding, as this is the only constraint that may prevent firms from reducing w_f.

Regarding (ii), it suffices to note that, in case of full employment, we would have $V_{uf} \simeq V_{ef} - \eta_w$. (NS_f) would thus amount to

$$V_{ef} \geq V_{ef} - \eta_w - \sigma_w + B_f > V_{ef} - \eta_w - \sigma_w + B,$$

contradicting Assumption A.

2. Characterization of the firing equilibrium

As before, we can solve for the wage that a firm has to pay to avoid shirking: using (7) and (8), and the binding (NS_f), the equilibrium values, which we label with the superscript "***", now satisfy the following labor supply condition:

$$w^{***} = \alpha_f + \beta_f v^{***}, \qquad (S_f)$$

where

$$\alpha_f \equiv w_u + e + (r + b + q_e)B_f - r\sigma_w > 0,$$

$$\beta_f \equiv (b + q_e)\left(B_f - \sigma_w - \eta_w\right) > 0,$$

where the inequalities follow from assumption A. The equilibrium values also satisfy the demand equation (6) which, using again $F_f'(L) = f'(\ell)$ and $v = \frac{L}{N-L}$, can be rewritten as:

$$F_f'\left(\frac{Nv^{***}}{1+v^{***}}\right) = w^{***} + \frac{1+r}{p_e}\rho + \frac{q_e + r}{p_e}\overline{w} + (b+q_e)\sigma_f + (b + q_e + r)\eta_f. \qquad (D_f)$$

The effects of parameter changes can be viewed through α_f and β_f, on the supply side; and through "mobility costs" $(b + q_e)\left(\sigma_f + \eta_f\right) + r\eta_f$ and "repair costs" $\frac{r+q_e}{p_e}\overline{w}$ on the demand side—worker mobility costs thus only affect the supply side, whereas firm mobility costs only affect the demand side. The qualitative analysis of comparative statics and welfare impacts follows much as in the no-firing case (see Appendix B).

In particular, increasing worker mobility costs has again positive general equilibrium effects on employment; as a result, it may still enhance total welfare if the elasticity of labor supply is large enough and the net productivity (net of effort and repair costs) is large enough. For example, the condition under which welfare is increased when worker hiring costs increase becomes:

$$\gamma^{***} > \left(\beta_f + \frac{1}{\lambda_f^{***}\varepsilon_f^{***}}\right)(1 + v^{***}),$$

which is satisfied if

$$\gamma^{***} > (1 + v^{***})\beta_f \text{ and } \varepsilon_f^{***} > \frac{1}{\left(\frac{\gamma^{***}}{1+v^{***}} - \beta_f\right)\lambda_f^{***}}.$$

A similar, slightly weaker condition ensures that welfare is increased when worker leaving costs increase.

3. Existence of the firing and no firing equilibria

Introducing a choice of technology (recuperating a machine through repair rather than having a worker man the machine) raises the question of which equilibrium is likely to arise. To address this issue, we have to ask: in the putative no firing equilibrium, would it ever pay a firm to fire and repair, and in the putative firing equilibrium, would it pay a firm to retain the worker at the ineffective unit?

Intuitively, the no-firing equilibrium cannot exist where the candidate equilibrium w_l is high – namely, higher than $\overline{w}$ – and firm firing costs are low. It would then pay any single firm to switch strategies, firing workers when a unit becomes ineffective, which in turn would generate general equilibrium effects – V_u would increase, as there would be greater flow into and out of the unemployment pool, and so other firms would have to raise wages at both effective and ineffective units. Specifically, the no-firing equilibrium exists if (and only if) the cost of repair $\overline{w}$ is sufficiently high relative to the cost of staffing an ineffective unit, w_l, namely, if (see Appendix C):

$$(\overline{w} - w_l^*) + (p_e + r)\sigma_f > b\big(\sigma_f + \eta_f\big). \tag{9}$$

That is, the direct savings plus the additional (discounted) mobility costs (associated with firing the worker when the unit becomes ineffective) have to be greater than the additional mobility costs associated with maintaining a worker at the ineffective unit.

The equilibrium wage w_l^* is, of course, an endogenous variable, a function of all the parameters, including the interest rate and mobility costs. However, $\overline{w}$ itself is a technological parameter, not dependent on any of these variables, so the condition for the existence of a no firing equilibrium may or may not be satisfied. From our earlier analysis:

$$w_l^* = w_u + e - B - \boldsymbol{r}\sigma_w + \beta v^*,$$

where it will be recalled that $\beta = b(B - \sigma_f - \eta_f)$. In particular, in the limiting case with no mobility costs, the existence of the no-firing equilibrium just depends on the relative value of $\overline{w}$ and w_l^*, where the latter is higher if the required bonus *B* is low (i.e., if there is little need for incentives to avoid shirking) and if labor productivity is high (so the average effective wage is high).

Conversely, if the economy is in the firing equilibrium, any firm could switch to a no-firing / no-quitting strategy as follows. When a unit becomes ineffective, it could offer the worker a contract, going forward, entailing a wage w_l so long as the unit remains ineffective, just high enough to induce the worker to stay.[41] The required wage depends on parameters such as the unemployment benefit w_u, the cost of effort e, or the level of employment (which affects the rate at which unemployed workers can find a job). Specifically, the required wage is (see Appendix C):

$$\hat{w}_l \equiv w_u + e - B - r\sigma_w + \beta_f v^{***},$$

where it can be recalled that $\beta_f = (b + q_e)(B_f - \sigma_w - \eta_w)$, with $B_f = B/p_e$.

Importantly, this required wage $\hat{w}_l$ can be less than the repair cost $\bar{w}$.[42] Furthermore, there can be further savings from reduced mobility costs, if the savings from firing and later subsequently hiring workers when a unit becomes ineffective are greater than the additional costs incurred from workers at ineffective units who leave and have to be replaced. As a result, the firing equilibrium exists only if:

$$\bar{w} - \hat{w}_l + p_e \eta_f \leq b(\sigma_f + \eta_f), \tag{10}$$

that is, if the direct savings from keeping the worker, together with the savings on mobility costs associated with not having to hire another worker when the unit will become effective, do not compensate for the mobility costs associated with maintaining a worker at the ineffective unit.

If conditions (9) and (10) are both satisfied, the firing and no firing equilibria can both exist. Conversely, if none of these conditions are satisfied, there will be neither a pure firing nor a pure no-firing equilibrium; in this case, a hybrid equilibrium can arise, with firms delegating the maintenance for a fraction of their ineffective units.

4. Comparison

We have now described two possible equilibria, one in which the firm does not fire workers (or pay a wage so low that they would be induced to quit) and another in which it fires workers with a bad signal. Under what conditions is each preferred from a social point of view? And will the market select the right form of equilibrium?

We have already provided the basic analytics required to answer these questions. There are

[41] Together with the equilibrium wage w^{***}, this wage also ensures that the worker is incentivized to exert effort, regardless of the current effectiveness of his unit.

[42] From our previous analysis, the effective wage $\hat{w}_f^{***}$ also increases with w_u.

significant macroeconomic externalities. Each firm takes the punishment provided by unemployment (which depends on both the unemployment rate and the rate of exit from unemployment) as given, when in fact those variables are endogenous. Each firm takes the variables determining the reservation wage and the participation wage as given, when in fact those variables are endogenous as well. And no firm takes into account the societal costs in providing w_u. Hence, there is no presumption that the market will be efficient, either in the choice of "regimes," or within the regime, in the level of wages offered. The market's selection of equilibrium depends on a quite different set of variables than that of a social planner maximizing societal welfare, and most importantly, on the additional government unemployment insurance subsidies that can be appropriated through firing.

Ignoring issues of distribution, the no-firing steady state has a higher net output if and only if:[43]

$$Mf\left(\frac{p_e L^*}{M}\right) - L^*\left[e + \rho + b(\sigma_f + \eta_f + \sigma_w + \eta_w) + \frac{1-p_e}{p_e}\overline{w}\right](\sigma_f + \eta_f + \sigma_w + \eta_w) - L^* e$$

$$> Mf\left(\frac{L^{***}}{M}\right) - L^{***}\left[e + \frac{\rho}{p_e} + (p_e + b)(\sigma_f + \eta_f + \sigma_w + \eta_w) + \frac{1-p_e}{p_e}\overline{w}\right].$$

There are four critical differences between the two states. First, the level of employment is different – as we have noted, the faster rate of turnover in the firing equilibrium would normally be expected to make L smaller, but this effect may be undone by differences in mobility costs. Second, turnover occurs only for exogenous reasons in the no-firing equilibrium, whereas in the firing equilibrium, it also occurs whenever an effective unit becomes ineffective – and in steady state, an equal number of ineffective units become effective, creating hiring costs. As a result, the turnover rate only depends on b in the no-firing equilibrium, whereas in the firing equilibrium it also depends on p_e. Third, in the firing equilibrium, there are the costs of repair $\overline{w}$.[44] Finally, in the no-firing equilibrium, the workers are spread over all units of production, but only a fraction p_e are effective, whereas in the firing equilibrium, workers are concentrated in the effective units. It is clear that the inequality may or may not hold, depending on the various mobility costs and on f''.

[43] Focusing on steady states ignores the timing of expenditures on mobility, but greatly simplifies the analysis.
[44] And the additional capital costs of unmanned ineffective units under repair.

V. Conclusion

This paper has served three purposes. Two of these relate to earlier literature. The first was to extend the Shapiro-Stiglitz analysis of the natural rate of unemployment by explicitly incorporating a wider range of instruments that might be used to deter shirking (beyond firing workers). The central result here was that, although firms may indeed employ wage reductions to deter shirking, participation constraints impose severe limitations on feasible wage reductions, and with these limitations, unemployment in general results. Indeed, all the central qualitative properties of the Shapiro-Stiglitz analysis remain valid. We have extended the welfare analysis, providing a fuller treatment of the consequences of various changes in parameters and policies both to societal welfare and the expected utility of an entrant into the labor force.

The second objective of the paper was to extend the standard principal-agent literature to a simple general equilibrium framework. In that literature, reservation utility levels play a critical role in ex ante and ex post participation constraints, with the latter setting limits on feasible punishments, which in turn give rise to worker rents. However, in the principal-agent literature, the reservation utility level is *exogenously given*, as are the participation constraints. But whether, for instance, an individual quits instead of accepting a punishment meted out, depends on the general equilibrium, and so too the reservation wage is determined as part of the general equilibrium. In this paper, we have made the reservation utility level endogenous, as well as the constraints on ex post participation. Of particular relevance (though not surprising) is the result that while the standard setup of the principal agent problem makes it seem as if the market solution is Pareto efficient, in general, the market equilibrium is not Pareto efficient.

The analysis of the role of mobility costs provided the third motive of this paper, and many of the central insights of the paper concern these costs. The nature of the equilibrium depends critically both on the costs faced by workers and those confronting firms. We had conjectured that with low mobility costs, punishments would be limited; and there would be welfare losses associated with the alternative methods of providing incentives—including the induced unemployment that might be necessary to provide workers with effective discipline. Our conjecture proved correct: an increase in "friction" – real costs of hiring and firing, when imposed on workers—might actually increase workers' welfare and reduce unemployment. Putting sand in the wheels can improve economic performance, at least as evaluated from the perspective of workers, even when there are real costs to that sand. Furthermore, artificially created frictions might serve as substitutes; and since they simply involved transfers, rather than real resources, welfare could be expected to be higher. However, we also showed that the sand had to be of the right type.

Our analysis seems to support two pieces of conventional policy wisdom—that employment frictions imposed on employers have adverse effects on unemployment, as do increases in unemployment benefits. But our analysis (like much of the conventional wisdom), in order to focus more clearly on the general equilibrium incentive effects, ignores other critical market failures.

Mobility and mobility costs have, in particular, effects which go beyond the particular issues examined here: with high mobility, firms may be less willing to invest in certain types of training, while high mobility costs may impede the effective matching of firms with workers (matching being relevant both for worker productivity as well as worker welfare arising from non-pecuniary job attributes). Most importantly, in a dynamic economy needing to reallocate labor constantly, an increase in mobility costs impedes such reallocations and makes them more costly.

Even in the absence of the considerations upon which we have focused here, we know that the equilibrium level of mobility that will emerge in market economies will not in general be optimal. Mobility is associated with important macroeconomic externalities.[45] The analysis of this paper has reinforced that conclusion: there is no presumption that the natural rate of unemployment which emerges in the market is optimal, and there accordingly exist welfare-improving government interventions.

Much of the policy discussion focusing on search and other incentive issues in labor markets not only gives short shrift to the externalities just noted, but also to the consequences of imperfect risk and capital markets, imperfect information and incomplete contracting, and monopsony and monopoly power. These market failures typically dominate, probably rightly so, the policy discourse for those defending labor market protections, and too often those advocating for weakening worker protections ignore these important dimensions. For instance, some argue that markets (under the pressure from unions) and governments (under the pressure from insecure workers) have led to job

[45] See Arnott and Stiglitz (1985) and Arnott, Hosios and Stiglitz (1987). Deli Gatti *et al* (2012) link these macroeconomic externalities associated with real impediments to labor mobility to deep economic downturns.

security programs providing *excessive* job protection, and that policy should be directed to reducing such protections. These policy discussions typically ignore the welfare (and even productivity) benefits from greater security—which obviously cannot be studied within the context of a model such as that presented here with linear utility functions (risk-neutral individuals). Mateen, Stiglitz, and Yun (2021) show the welfare benefits, for instance, of greater unemployment insurance under these conditions.[46] There are further welfare benefits which arise from curbing firms' ability to exploit workers in the presence of monopsony power, imperfect contracting, and asymmetries of information, including the enhanced efficiency and lower unemployment rates resulting from the ability to design better incentive structures, which this paper has demonstrated.

[46] While MSY emphasize the importance of incomplete risk and capital markets, incomplete contracting and the possibility of exploitation taking advantage of workers' imperfect information and shortsightedness is also important. Workers may not fully understand the extent of worker exploitation (abuse) that employers engage in; in the standard model with perfect rationality, workers would demand a compensating wage differential. Moreover, workers may be desperate for a job, accepting a job from a "bad" employer, feeling they have no choice. [The absence of good capital markets, giving rise to liquidity constraints, also plays an important role.]

Appendix

A. Welfare analysis for the no firing equilibrium

1. Workers' welfare

The equilibrium expected discounted utility of unemployed workers, V_u^*, satisfies (5). Differentiating with respect to w_u, β, and v and using the expression of dv^* then yields:

$$rdV_u^* = dw_u + v^* d\beta + \beta dv^*$$

$$= \frac{dw_u + v^* d\beta - \beta\lambda^*\varepsilon^*\left[d\alpha + \left(\sigma_f + \eta_f\right)db + bd\left(\sigma_f + \eta_f\right) + rd\eta_f\right]}{1 + \beta\lambda^*\varepsilon^*}.$$

The expected discounted utility of employed workers is $V_e \equiv p_e V_h + q_e V_l$. As (NB) and (NC) are both binding, in equilibrium this reduces to $V_e^* = V_u^* - \sigma_w + B$. Differentiating yields:

$$dV_e^* = dV_u^* + dB - d\sigma_w.$$

We can now use these results to analyze the impact on workers' welfare of a variety of changes in parameters and policies.

Friction costs. An increase in firms' exogenous mobility costs has no direct impact on workers but reduces firms' demand for labor, leading to lower wages and employment, and this indirect effect harms workers:

$$r\frac{\partial V_u^*}{\partial \sigma_f} = r\frac{\partial V_e^*}{\partial \sigma_f} = -\frac{\beta\lambda^*\varepsilon^*}{1 + \beta\lambda^*\varepsilon^*}b < 0,$$

$$r\frac{\partial V_u^*}{\partial \eta_f} = r\frac{\partial V_e^*}{\partial \eta_f} = -\frac{\beta\lambda^*\varepsilon^*}{1 + \beta\lambda^*\varepsilon^*}(b + r) < 0,$$

$$r\frac{\partial V_u^*}{\partial b} = r\frac{\partial V_e^*}{\partial b} = -\frac{\beta\lambda^*\varepsilon^*}{1 + \beta\lambda^*\varepsilon^*}\left(\sigma_f + \eta_f\right) < 0.$$

An increase in the hiring cost η_w reduces the slope β, which lowers workers' expected utilities (direct effect) but boosts employment (indirect effect); however, the direct effect dominates and, as a result, workers' (pre-transfers) expected levels of utility decrease:

$$r\frac{\partial V_u^*}{\partial \eta_w} = r\frac{\partial V_{\mathrm{e}}^*}{\partial \eta_w} = \left(r\frac{\partial V_u^*}{\partial \beta} + r\frac{\partial V_u^*}{\partial v^*}\frac{\partial v^*}{\partial \beta}\right)\frac{\partial \beta}{\partial \eta_w} = \left(v^* - \frac{\beta\lambda^*\varepsilon^* v^*}{1+\beta\lambda^*\varepsilon^*}\right)(-b) = -\frac{bv^*}{1+\beta\lambda^*\varepsilon^*} < 0.$$

An increase of the leaving cost σ_w has the same effect as an increase of η_w on the slope of the supply curve, but moreover translates the curve downwards, resulting in a bigger boost to employment (indirect effect) for the same overall effect on turnover costs (direct effect). It may even increase unemployed workers' (pre-transfers) expected utilities if labor demand is sufficiently elastic, with the benefit of the reduced unemployment now exceeding the losses from reduced wages:

$$r\frac{\partial V_u^*}{\partial \sigma_w} = \frac{\beta\lambda^*\varepsilon^* r - bv^*}{1+\beta\lambda^*\varepsilon^*},$$

implying that V_u^* increases if:

$$\varepsilon^* > \frac{bv^*}{\beta\lambda^* r}.$$

However, the impact of the switching cost σ_w on employed workers' welfare is unambiguously negative:

$$r\frac{\partial V_e^*}{\partial \sigma_w} = -\frac{bv^* + r}{1+\beta\lambda^*\varepsilon^*} < 0.$$

This is because an increase in σ_w has a direct impact on employed workers' outside options, as they would incur this cost upon leaving the firm.

Unemployment benefits. An increase in the unemployment benefits w_u shifts the supply curve upwards, leading to higher equilibrium wage and unemployment. Despite leading to a higher unemployment rate, it increases workers' expected utilities:

$$r\frac{\partial V_e^*}{\partial w_u} = r\frac{\partial V_u^*}{\partial w_u} = \frac{1}{1+\beta\lambda^*\varepsilon^*} > 0.$$

The direct impact of higher unemployment benefits thus more than offsets their undesirable effect on unemployment.

2. Total welfare

The equilibrium value of the total discounted industry profit, Π^*, satisfies:

$$r\Pi^* \equiv F\big(D(\widehat{w}_e^*)\big) - \big[w_e + \rho + b\big(\sigma_f + \eta_f\big)\big]D(\widehat{w}_e^*),$$

where $w_e \equiv p_e w_h + q_e w_l$ denotes the average wage paid to employed workers. Total welfare, defined as the sum of workers' payoffs and profits, net of the cost of financing unemployment benefits, satisfies:

$$\begin{aligned} rW &\equiv LrV_e + (N-L)rV_u + r\Pi - (N-L)w_u \\ &= F(L) + L\big[rV_e - w_e - b\big(\sigma_f + \eta_f\big)\big] + (N-L)(rV_u - w_u). \end{aligned}$$

Using (2), (3), (4) and $(N-L)\,v = L$, its equilibrium value is given by:

$$rW^* = F(L^*) - L^*\big[e + \rho + b\big(\sigma_w + \eta_w + \sigma_f + \eta_f\big)\big].$$

We thus have:

$$rdW^* = \frac{\gamma^* N}{(1+v^*)^2}dv^* - \frac{Nv^*}{1+v^*}\big[e + b\big(d\sigma_w + d\eta_w + d\sigma_f + d\eta_f\big) + \big(\sigma_w + \eta_w + \sigma_f + \eta_f\big)db\big],$$

where

$$\gamma^* \equiv F'(L^*) - e - \rho - b\big(\sigma_w + \eta_w + \sigma_f + \eta_f\big)$$

denotes the equilibrium productivity, net of the cost of effort and turnover, which is positive:

$$\begin{aligned} \gamma^* = {} & (1+r)\left[w_u + e + \tfrac{bB}{1+r} - r\sigma_w + \beta v + \rho + b\big(\sigma_f + \eta_f\big) + r\eta_f\right] \\ & -e - \rho - b\big(\sigma_w + \eta_w + \sigma_f + \eta_f\big) \\ = {} & (1+r)r\big(V_l + \eta_f\big) + r\big[e + \rho + b\big(\sigma_f + \eta_f\big)\big] + b[B - (\sigma_w + \eta_w)] \\ > {} & 0, \end{aligned}$$

where the first equality follows from (D) and (S), the second one from (5) and the – binding – (NB), and the inequality stems from Assumption A.

Friction costs. An increase in the friction costs faced by the firms depresses labor demand and reduces both wages and employment; as a result, welfare is also reduced. Indeed, we have:

$$r\frac{\partial W^*}{\partial \sigma_f} = -\frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*}b - \frac{Nv^*}{1+v^*}b < 0,$$

$$r\frac{\partial W^*}{\partial \eta_f} = -\frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*}(b+r) - \frac{Nv^*}{1+v^*}b < 0,$$

$$r\frac{\partial W^*}{\partial b} = -\frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*(\sigma_f+\eta_f)}{1+\beta\lambda^*\varepsilon^*} - \frac{Nv^*}{1+v^*}(\sigma_w+\eta_w+\sigma_f+\eta_f) < 0.$$

By contrast, the impact of the friction costs faced by the workers is less clear; we have:

$$r\frac{\partial W^*}{\partial \eta_w} = \left(\frac{\gamma^*}{1+v^*}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*} - 1\right)\frac{Nbv^*}{1+v^*}.$$

That is, an increase in the hiring cost η_w has a direct negative impact on welfare, but a positive indirect impact through higher employment. The overall effect is positive if:

$$\frac{\partial W^*}{\partial \eta_w} > 0 \Leftrightarrow \gamma^* > \gamma^*_{\eta_w} \equiv (1+v^*)\left(\beta + \frac{1}{\lambda^*\varepsilon^*}\right).$$

Alternatively, the condition can be expressed as $\gamma^* > \hat{\gamma}^*_{\eta_w} \equiv (1+v^*)\beta$ and $\varepsilon^* > \varepsilon^*_{\eta_w} \equiv \frac{1+v^*}{\lambda^*[\gamma^*-(1+v^*)\beta]}$

Likewise:

$$r\frac{\partial W^*}{\partial \sigma_w} = \left[\frac{\gamma^*}{1+v^*}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*}\left(1+\frac{r}{bv^*}\right) - 1\right]\frac{Nbv^*}{1+v^*} > r\frac{\partial W^*}{\partial \eta_w}.$$

In particular:

$$\frac{\partial W^*}{\partial \sigma_w} > 0 \Leftrightarrow \gamma^* > \gamma^*_{\sigma_w} \equiv (1+v^*)\left(\beta + \frac{1}{\lambda^*\varepsilon^*}\right)\frac{bv^*}{r+bv^*},$$

where $\gamma^*_{\sigma_w} = \frac{bv^*}{r+bv^*}\gamma^*_{\eta_w} < \gamma^*_{\eta_w}$. Alternatively, the condition can be expressed as $\gamma^* > \gamma^*_{\sigma_w} \equiv (1+v^*)\beta\frac{bv^*}{r+bv^*}$ and $\varepsilon^* > \varepsilon^*_{\sigma_w} \equiv \frac{bv^*(1+v^*)}{\lambda^*[\gamma^*(bv^*+r)-bv^*(1+v^*)\beta]}$, where $\hat{\gamma}^*_{\sigma_w} < \hat{\gamma}^*_{\eta_w}$ and $\varepsilon^*_{\sigma_w} < \varepsilon^*_{\eta_w}$.

Unemployment benefits. As we have seen, an increase in the unemployment benefits w_u leads to higher equilibrium wage and unemployment. Hence, despite increasing workers' expected utilities, it reduces total welfare once the cost of financing these unemployment benefits is accounted for:

$$\frac{\partial W^*}{\partial w_u} = \frac{\gamma^* N}{(1+v^*)^2}\frac{\partial v^*}{\partial w_u} < 0.$$

Policy determined separation / mobility costs. We have so far considered the case of *real* costs of mobility, but public policy can also affect mobility, and the revenues raised from taxes on firing or hiring, borne either by workers or firms, can be used for public purpose. Intuitively, such taxes will have a similar impact on wages and employment, but a different direct impact on welfare – it may remain negative due to distortive taxes or be positive if the collected resources are put to sufficiently good use.

To explore further the implications, suppose now that the hiring and leaving costs borne by the firms and the workers are pure redistribution transfers, with no direct impact on total welfare. The expressions of workers' expected utilities and firms' profits remain unchanged, as well as the characterization of equilibrium wages and employment, but total welfare is now such that:

$$r\widehat{W} \equiv LrV_e + (N - L)rV_u + r\Pi - (N - L)w_u + b\big(\sigma_w + \eta_w + \sigma_f + \eta_f\big),$$

and so its equilibrium value is given by:

$$r\widehat{W}^* = F(L^*) - L^*(e + \rho).$$

We thus have:

$$rd\widehat{W}^* = \frac{\hat{\gamma}^* N}{(1 + v^*)^2} dv^*,$$

where

$$\hat{\gamma}^* \equiv F'(L^*) - e - \rho$$

denotes the equilibrium productivity, net of the cost of effort and turnover, which is positive:

$$\begin{aligned}\gamma^* &= (1 + r)\left[w_u + e + \tfrac{bB}{1+r} - r\sigma_w + \beta v^* + \rho + b\big(\sigma_f + \eta_f\big) + r\eta_f\right] - e - \rho \\ &= (1 + r)\left[b\big(\sigma_f + \eta_f\big)\right] + r\left[(1 + r)\big(V_l + \eta_f\big) + e + \rho\right] + bB \\ &> 0,\end{aligned}$$

where the first equality follows from (D) and (S), the second one from (5) and the – binding – (NB), and the inequality stems from Assumption A. It follows that the impact of the mobility taxes on total welfare are the same as those on employment. An increase in worker mobility taxes therefore improves welfare, whereas an increase in firm mobility taxes decreases it. Indeed, we have:

$$r\frac{\partial \widehat{W}}{\partial \sigma_f} = -\frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*}b < 0,$$

$$r\frac{\partial \widehat{W}}{\partial \eta_f} = -\frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*}{1+\beta\lambda^*\varepsilon^*}(b+r) < 0,$$

but:

$$r\frac{\partial W^*}{\partial \eta_w} = \frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^* b v^*}{1+\beta\lambda^*\varepsilon^*} > 0,$$

$$r\frac{\partial W^*}{\partial \sigma_w} = \frac{\gamma^* N}{(1+v^*)^2}\frac{\lambda^*\varepsilon^*(r+bv^*)}{1+\beta\lambda^*\varepsilon^*} > 0.$$

B. Welfare analysis for the firing equilibrium

Differentiating (S_f) and (D_f) yields:

$$dv^{***} = -\frac{\lambda_f^{***}\varepsilon_f^{***}}{1+\beta_f\lambda_f^{***}\varepsilon_f^{***}}\begin{pmatrix} dw_u + \dfrac{q_e+r}{p_e}d\overline{w} + (b+q_e)d\sigma_f + (b+q_e+r)d\eta_f \\ -[r+(b+q_e)v^{***}]d\sigma_w - (b+q_e)v^{***}d\eta_w \\ +\left[B_f+\sigma_f+\eta_f+\left(B_f-\sigma_w-\eta_w\right)v^{***}\right]db \\ +[r+(b+q_e)(1+v^{***})]dB_f \end{pmatrix}, \quad (\mathrm{C.1})$$

$$dw^{***} = \frac{1}{1+\beta_f\lambda_f^{***}\varepsilon_f^{***}}\begin{pmatrix} dw_u - \beta_f\lambda_f^{***}\varepsilon_f^{***}\dfrac{q_e+r}{p_e}d\overline{w} \\ -\beta_f\lambda_f^{***}\varepsilon_f^{***}(b+q_e)d\sigma_f - \beta_f\lambda_f^{***}\varepsilon_f^{***}(b+q_e+r)d\eta_f \\ -[r+(b+q_e)v^{***}]d\sigma_w - (b+q_e)v^{***}d\eta_w \\ +\left[B_f(1+v^{***}) - \beta_f\lambda_f^{***}\varepsilon_f^{***}\left(\sigma_f+\eta_f\right) - (\sigma_w+\eta_w)v^{***}\right]db \\ +[r+(b+q_e)(1+v^{***})]dB_f \end{pmatrix}. \quad (\mathrm{C.2})$$

We thus have:

$$\frac{\partial v^{***}}{\partial w_u} < 0, \frac{\partial v^{***}}{\partial \overline{w}} < 0, \frac{\partial v^{***}}{\partial \sigma_f} < 0, \frac{\partial v^{***}}{\partial \eta_f} < 0, \frac{\partial v^{***}}{\partial \sigma_w} > 0, \frac{\partial v^{***}}{\partial \eta_w} > 0, \frac{\partial v^{***}}{\partial b} < 0, \frac{\partial v^{***}}{\partial B_f} < 0,$$

$$\frac{\partial w^{***}}{\partial w_u} > 0, \frac{\partial w^{***}}{\partial \overline{w}} < 0, \frac{\partial w^{***}}{\partial \sigma_f} < 0, \frac{\partial w^{***}}{\partial \eta_f} < 0, \frac{\partial w^{***}}{\partial \sigma_w} < 0, \frac{\partial w^{***}}{\partial \eta_w} < 0, \frac{\partial w^{***}}{\partial B_f} > 0,$$

and:

$$\frac{\partial w^{***}}{\partial b} > 0 \Leftrightarrow \varepsilon_f^{***} < \frac{B_f + \left(B_f - \sigma_w - \eta_w\right)v^{***}}{\beta_f \lambda_f^{***}\left(\sigma_f + \eta_f\right)}.$$

1. Workers' welfare

The equilibrium expected discounted utility of unemployed workers, V_{uf}^{***}, satisfies (8), which, using the binding $\left(\mathrm{NS}_f\right)$, yields:

$$rV_{uf}^{***} = w_u + \beta_f v^{***}. \tag{C.3}$$

Differentiating this expression yields:

$$rdV_{uf}^{***} = dw_u + v^{***}d\beta_f + \beta_f dv^{***},$$

which, using $(C.1)$, leads to:

$$rdV_{uf}^{***} = \frac{1}{1 + \beta_f \lambda_f^{***}\varepsilon_f^{***}}\begin{pmatrix} dw_u - \beta_f \lambda_f^{***}\varepsilon_f^{***}\dfrac{q_e + r}{p_e}d\bar{w} \\ -\beta_f \lambda_f^{***}\varepsilon_f^{***}(b + q_e)d\sigma_f - \beta_f \lambda_f^{***}\varepsilon_f^{***}(b + q_e + r)d\eta_f \\ -[r + (b + q_e)v^{***}]d\sigma_w - (b + q_e)v^{***}d\eta_w \\ +\left[\left(B_f - \sigma_w - \eta_w\right)v^{***} - \beta_f \lambda_f^{***}\varepsilon_f^{***}\left(B_f + \sigma_f + \eta_f\right)\right]db \\ -\beta_f \lambda_f^{***}\varepsilon_f^{***}(b + q_e + r)dB_f \end{pmatrix}.$$

We thus have:

$$\frac{\partial V_{uf}^{***}}{\partial w_u} > 0,\ \frac{\partial V_{uf}^{***}}{\partial \bar{w}} < 0,\ \frac{\partial V_{uf}^{***}}{\partial \sigma_f} < 0,\ \frac{\partial V_{uf}^{***}}{\partial \eta_f} < 0,\ \frac{\partial V_{uf}^{***}}{\partial \sigma_w} < 0,\ \frac{\partial V_{uf}^{***}}{\partial \eta_w} < 0,\ \frac{\partial V_{uf}^{***}}{\partial B_f} < 0,$$

and:

$$\frac{\partial V_{uf}^{***}}{\partial b} > 0 \Leftrightarrow \varepsilon_f^{***} < \frac{\left(B_f - \sigma_w - \eta_w\right)v^{***}}{\beta_f \lambda_f^{***}\left(B_f + \sigma_f + \eta_f\right)}.$$

Furthermore, the binding $\left(\mathrm{NS}_f\right)$ yields:

$$V_{ef}^{***} = V_{uf}^{***} - \sigma_w + B_f, \tag{C.4}$$

leading to:

$$rdV_{ef}^{***} = \frac{1}{1+\beta_f\lambda_f^{***}\varepsilon_f^{***}}\left(\begin{array}{c} dw_u - \beta_f\lambda_f^{***}\varepsilon_f^{***}\dfrac{q_e+r}{p_e}d\overline{w} \\ -\beta_f\lambda_f^{***}\varepsilon_f^{***}(b+q_e)d\sigma_f - \beta_f\lambda_f^{***}\varepsilon_f^{***}(b+q_e+r)d\eta_f \\ -\left[\left(2+\beta_f\lambda_f^{***}\varepsilon_f^{***}\right)r + (b+q_e)v^{***}\right]d\sigma_w - (b+q_e)v^{***}d\eta_w \\ +\left[\left(B_f-\sigma_w-\eta_w\right)v^{***} - \beta_f\lambda_f^{***}\varepsilon_f^{***}\left(B_f+\sigma_f+\eta_f\right)\right]db \\ -\left[r+\beta_f\lambda_f^{***}\varepsilon_f^{***}(b+q_e)\right]dB_f \end{array}\right).$$

We thus have

$$\frac{\partial V_{ef}^{***}}{\partial w_u} > 0, \frac{\partial V_{ef}^{***}}{\partial \overline{w}} < 0, \frac{\partial V_{ef}^{***}}{\partial \sigma_f} < 0, \frac{\partial V_{ef}^{***}}{\partial \eta_f} < 0, \frac{\partial V_{ef}^{***}}{\partial \sigma_w} < 0, \frac{\partial V_{ef}^{***}}{\partial \eta_w} < 0, \frac{\partial V_{ef}^{***}}{\partial B_f} < 0,$$

and:

$$\frac{\partial V_{ef}^{***}}{\partial b} > 0 \Leftrightarrow \varepsilon_f^{***} < \frac{\partial(B_f-\sigma_w-\eta_w)v^{***}}{\beta_f\lambda_f^{***}(B_f+\sigma_f+\eta_f)}.$$

2. Total welfare

The equilibrium value of the total discounted industry profit, Π^*, satisfies:

$$r\Pi^{***} \equiv F_f(L^{***}) - \left[w^{***} + \frac{q_e}{p_e}\overline{w} + \frac{\rho}{p_e} + (b+q_e)\left(\sigma_f+\eta_f\right)\right]L^{***}, \qquad (\mathrm{C}.5)$$

whereas total welfare satisfies:

$$rW^{***} \equiv L^{***}rV_{ef}^{***} + (N-L^{***})\left(rV_{uf}^{***} - w_u\right) + r\Pi^{***},$$

which, using $(\mathrm{C}.3)-(\mathrm{C}.5)$, yields:

$$rW^{***} = F_f(L^{***}) - L^{***}\left[e + \frac{q_e}{p_e}\overline{w} + \frac{\rho}{p_e} + (b+q_e)\left(\sigma_f+\eta_f+\sigma_w+\eta_w\right)\right].$$

We thus have:

$$rdW^{***} = \frac{\gamma^{***}N}{(1+v^{***})^2}dv^{***} - \frac{Nv^{***}}{1+v^{***}}\left[\begin{array}{c}(b+q_e)\left(d\sigma_w+d\eta_w+d\sigma_f+d\eta_f\right) \\ +\left(\sigma_w+\eta_w+\sigma_f+\eta_f\right)db + \dfrac{q_e}{p_e}d\overline{w}\end{array}\right],$$

where

$$\gamma^{***} \equiv F_f{}'(L^{***}) - \left[e + \frac{q_e}{p_e}\overline{w} + \frac{\rho}{p_e} + (b+q_e)\left(\sigma_f+\eta_f+\sigma_w+\eta_w\right)\right]$$

denotes the equilibrium productivity, net of the cost of effort and turnover. Using (D_f) leads to:

$$\gamma^{***} = w_u + \frac{r}{p_e}\overline{w} + \frac{r}{p_e}\rho + r\eta_f + (b + q_e)\big(B_f - \sigma_w - \eta_w\big)(1 + v^{***}) + r\big(B_f - \sigma_w\big) > 0,$$

where the inequality follows from Assumption A.

We thus have:

$$\frac{\partial W^{***}}{\partial w_u} < 0, \frac{\partial W^{***}}{\partial \overline{w}} < 0, \frac{\partial W^{***}}{\partial \sigma_f} < 0, \frac{\partial W^{***}}{\partial \eta_f} < 0, \frac{\partial W^{***}}{\partial b} < 0, \frac{\partial W^{***}}{\partial B_f} < 0,$$

and:

$$r\frac{\partial W^{***}}{\partial \eta_w} = \frac{\gamma^{***}N}{(1 + v^{***})^2}\frac{\lambda_f^{***}\varepsilon_f^{***}}{1 + \beta_f\lambda_f^{***}\varepsilon_f^{***}}(b + q_e)v^{***} - \frac{N(b + q_e)v^{***}}{1 + v^{***}},$$

$$r\frac{\partial W^{***}}{\partial \sigma_w} = \frac{\gamma^{***}N}{(1 + v^{***})^2}\frac{\lambda_f^{***}\varepsilon_f^{***}}{1 + \beta_f\lambda_f^{***}\varepsilon_f^{***}}[r + (b + q_e)v^{***}] - \frac{N(b + q_e)v^{***}}{1 + v^{***}}.$$

Therefore:

$$\frac{\partial W^{***}}{\partial \eta_w} > 0 \Leftrightarrow \gamma^{***} > \left(\beta_f + \frac{1}{\lambda_f^{***}\varepsilon_f^{***}}\right)(1 + v^{***}),$$

which can be expressed as:

$$\gamma^{***} > (1 + v^{***})\beta_f \text{ and } \varepsilon_f^{***} > \frac{1}{\left(\frac{\gamma^{***}}{1 + v^{***}} - \beta_f\right)\lambda_f^{***}}.$$

Likewise:

$$\frac{\partial W^{***}}{\partial \sigma_w} > 0 \Leftrightarrow \gamma^{***} > \left(\beta_f + \frac{1}{\lambda_f^{***}\varepsilon_f^{***}}\right)\frac{1 + v^{***}}{1 + \frac{r}{(b + q_e)v^{***}}},$$

which can be expressed as:

$$\gamma^{***} > \frac{(1 + v^{***})\beta_f}{1 + \frac{r}{(b + q_e)v^{***}}} \text{ and } \varepsilon_f^{***} > \frac{1}{\left\{\frac{\gamma^{***}}{1 + v^{***}}\left[1 + \frac{r}{(b + q_e)v^{***}}\right] - \beta_f\right\}\lambda_f^{***}}.$$

C. Existence of the firing and no firing equilibria

Starting from the no-firing equilibrium characterized in Section III, if a firm were to deviate and fire a worker when the unit becomes ineffective, the marginal profitability of that ineffective unit (gross of the cost of firing the worker), $\hat{P}_l^*$, would satisfy:

$$\hat{P}_l^* = -\overline{w}t + \exp(-rt)\big[p_e t\big(P_h^* - \hat{P}_l^*\big) + \hat{P}_l^*\big],$$

which, using $\exp(-rt) \simeq 1 - rt$, amounts to:

$$(r + p_e)\hat{P}_l^* = -(\overline{w} + \rho) + p_e P_h^*, \tag{D.1}$$

where P_h^* is the productivity of effective units in the no-firing equilibrium, which from our previous analysis (see footnote 28) is given by:

$$P_h^* = \frac{w_l^* + \rho + b\sigma_f + (b + r + p_e)\eta_f}{p_e}. \tag{D.2}$$

To ensure that this deviation is not profitable, we must have $\hat{P}_l^* - \sigma_f \leq P_l^* = \eta_f$ or, using $(\mathrm{D.1})$ and $(\mathrm{D.2})$:

$$(\overline{w} - w_l^*) + \ (p_e + r)\sigma_f \geq b\big(\sigma_f + \eta_f\big).$$

Conversely, starting from the firing equilibrium characterized in Section IV, suppose that a firm deviates and seeks to keep a worker, with a (lower) wage w_l, when his unit becomes ineffective. Accepting to stay at that wage gives the worker an expected utility such that:

$$V_l = (w_l - e)t + \exp(-rt)\big\{bt\big(V_{uf}^{***} - \sigma_w\big) + q_e t V_{ef}^{***} + [1 - (b + q_e)t]V_l\big\},$$

which, using $\exp(-rt) \simeq 1 - rt$, the binding $\big(\mathrm{NS}_f\big)$ and the definition of B_f and B, yields:

$$(r + b + p_e)V_l = w_l - e + b(b + q_e)\big(V_{uf}^{***} - \sigma_w\big) + B.$$

To induce the worker to stay, this expected utility must be at least equal to $V_{uf}^{***} - \sigma_w$; the minimal acceptable wage, $\hat{w}_l$, is therefore such that:

$$\hat{w}_l - e + (b + q_e)\big(V_{uf}^{***} - \sigma_w\big) + B = (r + b + p_e)\big(V_{uf}^{***} - \sigma_w\big),$$

leading to:

$$\widehat{w}_l = e - B + r\left(V_{uf}^{***} - \sigma_w\right) = w_u + e - B - r\sigma_w + \beta_f v^{***},$$

where the last equality stems from (C.3). By construction, as this wage gives the worker the same expected utility as from leaving the firm (i.e., $V_l = V_{uf}^{***} - \sigma_w$), it also maintains the incentive to exert effort.[47]

The resulting marginal profitability for the ineffective unit satisfies:

$$\widehat{P}_l^{***} = -\left[\widehat{w}_l + \rho + b\left(\sigma_f + \eta_f\right)\right]t + \exp(-rt)\left[p_e P_{hf}^{***} + (1 - p_e t)\widehat{P}_l^{***}\right],$$

which, using again $\exp(-rt) \simeq 1 - rt$, amounts to:

$$(r + p_e)\widehat{P}_l^{***} = -(\widehat{w}_l + \rho) - b\left(\sigma_f + \eta_f\right) + p_e P_{hf}^{***}, \tag{D.3}$$

where P_{hf}^{***} is the productivity of effective units in the firing equilibrium, which from our previous analysis (see footnote 40) is given by:

$$P_{hf}^{***} = \frac{\overline{w} + \rho}{p_e} + \eta_f. \tag{D.4}$$

To ensure that this deviation is not profitable, we must have $\widehat{P}_l^{***} \leq P_{lf}^{***} = 0$, or, using $(\mathrm{D.3})$ and $(\mathrm{D.4})$:

$$\overline{w} - \widehat{w}_l + p_e\eta_f \leq b\left(\sigma_f + \eta_f\right).$$

[47] In the firing equilibrium, the no-shirking constraint $\left(\mathrm{NS}_f\right)$ amounts to $V_{ef}^{***} - \left(V_{uf}^{***} - \sigma_w\right) \geq B_f$. It therefore also ensures that the no-shirking condition (NS), which amounts to $V_h - V_l \geq B_f$, holds for $V_h = V_{ef}^{***} - \sigma_w$ and $V_l = V_{uf}^{***} - \sigma_w$.

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