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#### ARE SUPPLY NETWORKS EFFICIENTLY RESILIENT?

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### **ABSTRACT**

We show that supply networks are inefficiently, and insufficiently, resilient. Upstream firms can expand their production capacity to hedge against supply and demand shocks. But the social benefits of such investments are not internalized due to market power and market incompleteness. Upstream firms under-invest in capacity and resilience, passing-on the costs to down-stream firms, and drive trade excessively towards the spot markets. There is a wedge between the market solution and a constrained optimal benchmark, which persists even without rare and large shocks. Policies designed to incentivize capacity investment, reduce reliance on spot markets, and enhance competition ameliorate the externality.

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The shortages and spikes in prices of certain intermediate goods during the pandemic demonstrated the fragility of supply chains. Prominent examples include a global shortage of semi-conductors leading to a dramatic rise in the price of secondhand cars in the U.S.; and an unprecedented demand for hand sanitizers and personal protective gear triggering supply shortages in their respective, as well as interlinked, industries. Policymakers reacted strongly by taking industry-specific actions to improve resilience and repair linkages. For example, the Biden-Harris Administration worked in partnership with Congress to provide new legislation to alleviate specific supply chain disruptions and promote greater resilience in future situations. Moreover, while the large and small supply chain disruptions during Covid-19 had propelled the issue into popular discourse, the cracks had been evident before the pandemic. Hanjin Shipping, a world's top ten container carrier, filed for bankruptcy in September 2016 due to sluggish freight rates caused by weak demand and soaring global capacity. The bankruptcy affected global supply chains, because half of Hanjin's container ships were denied access to ports. Major U.S. retailers, such as J.C. Penney and Walmart, began to divert and switch carriers for their containers to other suppliers. Similarly, the failure of Carillion in January 2018, once the second largest construction company in the U.K., brought down many of its suppliers.<sup>1</sup>

These experiences with supply network disruptions left open the question: had firms invested too little in resilience ex ante? The pandemic was an extreme event, and in general firms should not be expected to anticipate and to plan for every possible contingency. Doing so would almost surely be inefficient, entailing excessive focus on resilience. We show here however, that given market power and market incompleteness, one should expect markets to under-invest in resilience relative to a constrained efficient benchmark.

We formulate a tractable theoretical model whereby a collection of intermediate and final good producers form supply linkages to meet uncertain consumer demand

<sup>&</sup>lt;sup>1</sup>See FT: Car chip shortage shines light on fragility of US supply chain; CNN: Distilleries are making hand sanitizers with their in-house alcohol and giving it out for free to combat coronavirus; Bidden-Harris Supply Chain Disruption Task Force; DW: Bankrupt Hanjin sparks shipping crisis and Guardian: Carillion collapse: two years on, the government has learned nothing". for detailed coverage of these episodes. A more academic account can be found in Baqaee and Farhi (2022), Guerrieri et al. (2022) and Di Giovanni et al. (2022).

and accommodate supply shocks. Each final good producer (the downstream firm) can source differentiated inputs from one or more suppliers of the intermediate good (the upstream firms). Intermediate good producers engage in price (i.e. Bertrand) competition with differentiated products, taking the prices set by competitors as given. Lowering the price charged allows an intermediate good producer to increase demand on the extensive margin (by attracting more final good producers).<sup>2</sup> Intermediate good producers face uncertainty in demand and supply conditions. They invest in non-scalable production capacity before the realization of shocks, reflecting the fact that some factors of production cannot be readily adjusted at short notice.<sup>3</sup> Given the structural frictions in the economy, namely the lags in production and the uncertainty around future market conditions, over-investment in capacity can be just as inefficient as under-investment. A supply network that is *efficiently resilient* strikes the optimal balance on resilience, taking into account these structural frictions.

Using the model, we demonstrate the existence of a market failure in decentralized supply networks, whereby upstream firms do not fully internalize the social benefits of building production capacity. When upstream firms over-invest in capacity, part of the cost savings are passed-on to downstream firms via lower prices; but when firms under-invest, they can defend their profit margins in spite of mounting costs by charging higher prices. The shortages that result from under-investments enhance market power, which the upstream firms rationally anticipate. As a result, upstream firms will always lean towards under-investment.

This pecuniary externality is not internalized by the decentralized market due to a combination of (1) market power and (2) market incompleteness. First, upstream intermediate good producers exhibit market power because: (a) there are only a finite number of such firms; and (b) the intermediate goods they produce are imperfect substitutes of each other. Second, firms do not have access to the full set of Arrow-Debreu securities, and instead must trade either on the pre-order market, or on the spot market once the shocks have realized. The pre-order market offers par-

<sup>&</sup>lt;sup>2</sup>In a more general case, lowering the price may also affect the intensive margin.

<sup>&</sup>lt;sup>3</sup>Semiconductors is an example of an important intermediate good that requires significant capacity investment upfront. In the EU, The European Chips Act (2023) aims to provide additional public and private investments of more than EUR 15 billion.

tial insurance to both the upstream and downstream firms. For the upstream firms, pre-orders establish a minimum level of demand for their outputs, and help with their upfront non-scalable capacity investment decision. For the downstream final good producers, a pre-order contract locks in an agreed price for the intermediary inputs in their production, shielding them from cost shocks in the upstream sector. If realized demand for final goods exceeds what can be fulfilled through pre-orders, the downstream firm can then source the extra inputs required from the spot market. As we observe in practice, the spot and pre-order markets are insufficient to deal with the full spectrum of possible shocks, and thus unable to provide full insurance against supply network disruptions.<sup>4</sup>

Taken together, we show that the market-based network invests too little in production capacity  $(K^*)$  relative to a constrained optimal benchmark  $(K^{SP})$  with a social planner facing the same informational and technological constraints as the private market. Even under the constrained benchmark, it is not optimal to build enough capacity to account for all contingencies. So there will be times when firms ex post have considerable market power, which, obviously, the social planner would not take advantage of but private firms would. In short, market based supply networks are *inefficiently resilient*:  $K^* < K^{SP}$ .

Remarkably, this wedge between the decentralized and centralized solution arises even when rare large shocks are absent, and the economy operates in a "full production" equilibrium whereby supply is sufficiently agile to accommodate all possible demand. Our results do not depend on an arbitrary specification of the distribution of shocks (e.g. we do not require a threshold for the probability of large negative shocks). Nor do we need to impose a level of risk aversion on the part of private agents or social planner. Capacity investment is sub-optimally low, even when every agent – including the constrained social planner – is risk-neutral.

Extending the analysis to account for rare disasters (in the appendix), we show that the response of market-based supply networks to shocks can be highly non-

<sup>&</sup>lt;sup>4</sup>It is obvious that such full insurance *does not* exist. Given the range of shocks that could occur – some of which are now not even really conceivable – the incompleteness of insurance markets is inevitable. Theories of asymmetric information provide further explanations of the absence of a full set of insurance markets. See Greenwald and Stiglitz (1986) and Stiglitz (1982).

linear. Private supply networks are seemingly resilient<sup>5</sup> during normal times and can comfortably withstand small to moderate shocks, but are fragile to rare large shocks, when real rigidities prevent suppliers from fully meeting the needs of the market. With a large enough shock, there is a transition from a monopolistically competitive regime to a local monopoly regime – whereby upstream firms are no longer pricing to compete and each downstream firms will only receive one credible offer for inputs. In other words, in a crisis, individual suppliers prioritize the needs of their local market but with increased margins.<sup>6</sup> Supply network fragility can lead to an increase in market power (in our model, reflected in suboptimal retrenchment in market coverage), especially when demand is at its greatest.

The size of the wedge between the decentralized and centralized solution depends endogenously on firms' reliance on the spot market, and exogenously on the structural parameters of the economy. An economy exhibiting greater scalability (production functions that rely less on non-scalable capacity investments), higher substitutability (intermediate good inputs that are more inter-changeable) and more competition (more upstream firms) will be more efficiently resilient.

Therefore, there are broadly three avenues for narrowing the wedge. First, a direct governmental subsidy targeting investment in production capacity could serve as the most pragmatic remedy. Second, enhancing incentives for the use of preorder markets can offer upstream firms the assurance of recouping initial costs. We show that an over-reliance on the spot market contributes to fragility in the supply network. Lastly, the government can promote structural changes in the economy to enhance scalability, substitutability and competition. Enhancing competition is

<sup>&</sup>lt;sup>5</sup>By "seemingly resilient", we mean that demand can be fully met *at some price*. It is still the case that there is too little capacity.

<sup>&</sup>lt;sup>6</sup>The surge in demand for Covid vaccines in 2021 and the frantic pursuit of natural gas during the European energy crisis in 2022 serve as illustrative examples. Global supply constraints often lead to redirection towards wealthier nations, leaving less affluent developing markets economically disadvantaged during challenging times. During the post-Covid recovery, there is evidence of a marked increase in market power (markups) associated with the supply chain interruptions. See Konczal and Lusiani (2022).

<sup>&</sup>lt;sup>7</sup>For instance, in 2021 and 2022, more than 30 energy companies in the UK failed due to a rapid increase in wholesale natural gas prices and inadequate hedging through futures/forward contracts by the energy companies. See https://www.forbes.com/uk/advisor/energy/failed-uk-energy-suppliers-update/ for details.

good in its own right, and doubly so when making supply networks more efficiently resilient.

### 1 Related Literature

The literature on the resilience of supply networks to shocks can be roughly categorized into two branches. The first focuses on analyzing the mechanisms through which idiosyncratic shocks propagate and amplify within a fixed network of firms with pre-specified relationships. Acemoglu and Tahbaz-Salehi (2020) examine the impact of productivity shocks on the distribution of economic surplus, firm failures, and the amplification of shocks through disruptions. Acemoglu et al. (2012) propose a model that explains how micro shocks can be magnified into macro fluctuations through input-output linkages. Carvalho et al. (2021) use data from the 2011 Japanese earthquake to demonstrate the significant macroeconomic implications of idiosyncratic shocks. Barrot and Sauvagnat (2016) reveal evidence of fragility caused by the propagation of firm-specific shocks, using data on natural disasters. We refer to Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for a thorough review of such mechanisms.

That markets wouldn't be prepared for *every* shock they confront is not a surprise. The analytically interesting question is the normative one: relative to an appropriate benchmark, do they adequately prepare for shocks? The failure of each firm in a competitive environment to take account of how capacity decisions affect the distribution of prices in the spot markets is one of the two central market failures that we identify.

The second branch of literature focuses on firms' strategic responses to mitigate the negative impacts of supply chain disruption. Birge et al. (2023) explore how firms in a supply chain network strategically react post-disruption by optimally switching demand and rerouting supply from defaulted firms. Amelkin and Vohra (2020) examine the competing retailers' decision-making process when selecting suppliers, taking into account factors such as prices and suppliers' reliability as measured by yield uncertainty and congestion.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup>A few other studies from the operations management literature analyze the mechanisms through

Our work is closely related to that by Elliot et al. (2022) and Grossman et al. (2023) which (also) examine supply network formation and fragility. In their models, downstream firms source customized inputs from upstream firms. To insure against possible supply disruptions, downstream firms strategically invest in relationships with multiple potential suppliers. One might infer from their analyses that systemic fragility should be reduced if inputs were more (albeit still imperfectly) substitutable, and there existed a common spot market for such inputs. We show that such a spot market would not only be insufficient to eliminate supply network fragility, but an over-reliance on spot market transactions by market participants would actually amplify the inherent externalities. In our model, fragility within the supply network is not a consequence of a catastrophic break-down of upstream suppliers or a failure in supplier diversification, but due to a more structural combination of market power and incomplete markets.

On the empirical side, Atalay et al. (2011) estimate a model of firms' buyer-supplier relationships using microdata on firms' customers. Crosignani et al. (2019) investigate the consequences of supply shocks resulting from NotPetya, one of the most severe cyberattacks in history. They observe that the affected downstream customers were more inclined to establish new relationships with alternative suppliers while terminating existing relationships with the directly affected firms. Lastly, Baldwin and Freeman (2022) examines the cross-border dimensions of resilience in global supply chains.

The rest of the paper is organized as follows. Section 2 sets-up the model economy. Section 3 constructs the social planner benchmarks, and characterizes the constrained-optimal level of capacity investment ( $K^{SP}$ ). Section 4 characterizes the decentralized equilibrium and the market solution for capacity ( $K^*$ ). Section 5 presents our core result that firms' investment in capacity is insufficient:  $K^* < K^{SP}$  and discusses potential policy interventions. Section 6 concludes with suggestions for further research. Detailed derivations and proofs are found in the appendix, along with an extension of the analysis to rare large shocks pushing the economy

which multi-sourcing strategies and supplier selection can help mitigate risk in supply chains. See Anupindi and Akella (1993), Tomlin (2006), Babich et al. (2012) and Babich et al. (2007).

<sup>&</sup>lt;sup>9</sup>See also Elliott and Golub (2022) for a survey on supply chain disruptions and their macroeconomic implications.

away from full production.

### 2 Model

Consider an economy with two types of goods: final goods (the consumption numeraire) and intermediate goods used in the production of the final goods. There are a continuum of final good producers (i.e., downstream firms, indexed  $i \in I = [0,1]$ ) and  $n \geq 2$  intermediate good producers (i.e., upstream firms, indexed  $j \in J = \{0,1,\ldots,n-1\}$ ), all located around a circle with unit circumference. The positions of the intermediate good producers around the circle are represented by nodes, which divide the continuum of final good producers into n "market segments". Figure 2.1 illustrates a simplified example of such an economy with n=3 intermediate good producers. Distance is quantified along the circle's circumference, ensuring that the maximum distance separating any two points is  $\frac{1}{2}$ .

j = 0  $I_{2}$   $I_{1}$   $I_{2}$ 

Figure 2.1: Illustrative Economy

Consider an illustrative economy with three intermediate good firms ( $j \in \{0,1,2\}$ ). The intermediate good firms are located equi-distant from each other, separating the circle into three equal market segments  $\{I_0, I_1, I_2\}$ . In a typical equilibrium, Firms j = 0 and j = 1 compete over final good firms located in the market segment  $I_0$ .

Intermediate good producers  $j \in J$  are price-setters. They set prices  $\{p_j\}$  to compete over final good producers in their two neighboring market segments.<sup>10</sup> The

<sup>&</sup>lt;sup>10</sup>It is possible for any particular intermediate good producer to price so aggressively as to capture

mass of final good producers in each market segment are denoted as  $\{m_k\}_{k=0,\dots,n-1}$ . To fulfill the endogenous demand for intermediate goods, each intermediate good producer j operates a Cobb-Douglas production function with partial delay:  $Y_{j,t} = L_{j,t}^{\alpha_j} K_{j,t-1}^{1-\alpha_j}$ , where  $L_j$  denotes the scalable input factors in production with factor price  $w_j > 0$ ; and  $K_j$  the non-scalable capacity investments that must be installed one period in advance at unit price  $r_j > 0$ . The key distinction is that non-scalable inputs  $K_j$  cannot be adjusted in the short-run. The parameter  $\alpha_j \in (0,1)$ , the exponent of L, measures the *scalability* of each sector j. Crucially, intermediate good producers j must decide on the level of non-scalable capacity investments  $K_j$  before the realization of shocks to the economy. As we will discuss in greater detail below, the intermediate good producer's capacity investment  $(K_j)$ , and pricing decisions (on both the spot and future market) form the core of our model.

We model the final good producers in a more reduced-form fashion. Specifically, final good producers  $i \in I$  are price-takers. Each atomistic final good producer i faces an exogenous demand  $Q_i$  for their output, valued at unit price v.<sup>12</sup> They convert intermediate goods into the final good using a linear production function  $\tilde{Y}_i = \sum_j \frac{1}{f(d(i,j))} q_{ij}$ , where  $\tilde{Y}_i$  denotes the final good output of firm i;  $q_{ij}$  is the quantity of intermediate good input firm i sources from firm j; and f(d(i,j)) is a penalty function that depends on the distance  $(d(i,j) \in [0,\frac{1}{2}])$  between the two firms.

demand from market segments further afield. This corresponds to the "super-competitive" region of the demand curve in a circular economy (see Salop (1979)). For the purpose of the present analysis, our closed-form solutions focus on a symmetric equilibrium in which all intermediate good producers find it optimal to set the same price, thus ruling out competition outside of the neighboring market segments.

<sup>&</sup>lt;sup>11</sup>For brevity we will henceforth drop the time subscripts, and note simply that K must be precommitted in advance of production.

 $<sup>^{12}</sup>$ In our model, final good firms form expectations over the level of demand  $Q_i$  taking the price v as a fixed constant; whereas more generally, shocks to final good demand would affect both (their desired) equilibrium quantity  $Q_i$  and price  $v_i$ . We simplify the analysis by taking the integral over the distribution of  $Q_i$  only, instead of the joint distribution over both  $Q_i$  and  $v_i$ . This simplifications offers greater analytical tractability, highlights the critical market failures, whilst preserving the essential economics of resilience. One can think of this either as: (1) a stylized portrayal of final good demand - a demand curve with demand equal to Q for price equal or less than v, and zero demand for price above v; or (2) a description of specific markets - like that for electricity - in which all firms have signed contracts to deliver output at price v regardless of the level of demand that materialize.

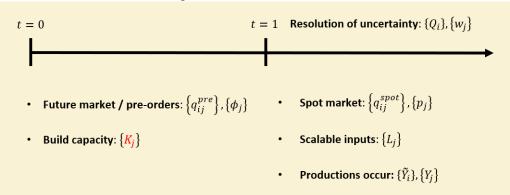
One way to think about this distance-based penalty function is that for every unit of intermediate good j purchased by i, only a fraction  $\frac{1}{f(d(i,j))}$  is usable. The remainder,  $\left(1 - \frac{1}{f(d(i,j))}\right)$ , "perishes in transit". Another interpretation of  $f(\cdot)$  is a valuation-based penalty function. For any given valuation v, the effective valuation of the final good i that uses inputs j is given by  $\frac{v}{f(d(i,j))}$ . Therefore, the function  $f(\cdot)$  can also account for heterogeneous valuations of final goods. Specifically, a final good firm i producing outputs using more "distant" intermediate goods would experience a diminished valuation for their output. A third interpretation (and the one we focus upon in the discussion below) is that the different intermediate goods are imperfect substitutes for each other. The production at any place in the circle is designed for a certain type of intermediate good, but can use other intermediate goods, though they yield less output per unit of input. (Think of an oil refinery designed to refine oil of a specific gravity and sulfur content. It can refine oil with other characteristics, but less efficiently). For ease of exposition, we will refer to  $f(\cdot)$  henceforth as the distance-based penalty function (distance, in this interpretation, refers to distance in the product space).<sup>13</sup>

We assume that  $f(\cdot)$  is an increasing function, normalized such that f(0) = 1. This penalty function  $f(\cdot)$ , combined with the starting distance between firms d(i,j), captures the extent of *substitutability* among intermediate goods. The greater the distance d(i,j) between two firms, and the steeper the slope f'(x) of the penalty function, the more inefficient it is for final good producer i to source inputs from intermediate good producer j. For brevity, let  $f_{ij} := f(d(i,j))$ ; and  $\mathbf{f}_i := (f_{i0}, \dots, f_{i,n-1})'$  be the corresponding  $n \times 1$  column vector of penalties for final good producer i.

Figure 2.2 summarizes the timeline of the model. At period 0, there is uncertainty around the demand and supply conditions that will prevail in period 1. Specifically, the uncertainty around the demand for final goods produced by firm i is captured by the random variable  $Q_i$ .  $Q_i$  is distributed between  $\left[\underline{Q}_i, \bar{Q}_i\right]$ , with cumulative density function (c.d.f.)  $G_i(\cdot)$  and associated probability density function (p.d.f.)  $g_i(\cdot)$ . There is also uncertainty around  $\{w_j\}_{j\in J}$ , the price of the scalable input factor, which affects the supply of the intermediate good j.  $w_j$  is distributed

<sup>&</sup>lt;sup>13</sup>For a discussion of the measurement of distance in product space, see, e.g., Stiglitz (1986).

Figure 2.2: Model Timeline



Timeline of events, decisions and actions undertaken by intermediate and final good producers.

between  $[\underline{w}_j, \overline{w}_j]$ , with c.d.f.  $H_j(\cdot)$  and p.d.f.  $h_j(\cdot)$ .<sup>14</sup> Supply shocks are assumed to be independent of demand shocks. In our formulation, there is no uncertainty about the price of the final good - it is the numeraire.

In period 0, to hedge against these demand and supply shocks, each final good producer i decides whether to enter into a supplier contract with each intermediate good producer j, placing pre-orders  $\mathbf{q}_i^{pre} \coloneqq \left[q_{i0}^{pre}, \ldots, q_{ij}^{pre}, \ldots, q_{in-1}^{pre}\right]'$ . Each intermediate good producer j sets pre-order price  $\phi_j$ . Concurrently, they make a cost-minimizing decision on the level of non-scalable capacity  $K_j$ , incurring associated costs denoted by  $r_jK_j$ . The pre-order contracts between final good and intermediate good producers define the *endogenous* network formed in period 0.

In period 1, firms observe the realization of the demand and supply shocks. Final good producer i submits spot-market orders  $\mathbf{q}_i^{spot} := \left[q_{i0}^{spot}, \ldots, q_{ij}^{spot}, \ldots, q_{in-1}^{spot}\right]'$ . The total cost of pre-orders and spot-market orders for firm i is given by  $\left[\phi \cdot \mathbf{q}_i^{pre} + \mathbf{p} \cdot \mathbf{q}_i^{spot}\right]$ , where  $\phi := \left[\phi_0, \ldots, \phi_j, \ldots \phi_{n-1}\right]'$  denote the vector of pre-order prices, and  $\mathbf{p}$  the vector of spot-market prices. At period 1, intermediate good producer j takes precommitted capacity  $K_j$  as given, solves for the cost-minimizing scalable input  $L_j$ , and sets prices  $p_j$  to maximize profits. Production occurs and contracts are settled. The excess of production over the contracted pre-orders is sold on the spot market.

In our model, the final good producers can buy from any intermediate good

<sup>&</sup>lt;sup>14</sup>Without loss of generality, let  $\infty > \bar{Q}_i > \underline{Q}_i > 0$ ,  $\forall i \in [0,1]$ ; and  $\infty > \bar{w}_j > \underline{w}_j > 0$ ,  $\forall j \in J$ .

producer at the posted price. This stands in contrast to much of the network literature discussed in the previous section (e.g. Elliot et al. (2022)), where final goods producers can only buy from the firms with whom they have previous relations, so shocks to those firms obviously get passed on strongly through the network. Here, in effect, the ex ante and ex post networks can be different. We assume that there are no costs to establishing a new link ex post.<sup>15</sup>

For analytical tractability, we impose symmetry on the primitives of the model and derive closed form solutions for the resulting symmetric equilibrium.

**Assumption** A1 [**Symmetry**]: 
$$\alpha_j = \alpha$$
 and  $r_j = r$ ,  $\forall j \in J$ ;  $Q_i = Q$ ,  $\forall i \in I$ ;  $w_j = w$ ,  $\forall j \in J$ ;  $m_k = \frac{1}{n}$ ,  $\forall k \in \{0, \dots, n-1\}$ 

By assumption, all intermediate good producers share a common Cobb-Douglas production function:  $\alpha_j = \alpha, \forall j \in J$ ; and face the same non-scalable input costs in period 0:  $r_j = r, \ \forall j \in J$ . We also assume that the shocks to the economy are symmetric and identical. The realization of final good demand is the same for all final good firms:  $Q_i = Q, \ \forall i \in I$ ; and the realization of scalable input cost is also the same for all intermediate good firms:  $w_j = w, \ \forall j \in J$ . This symmetry captures an economy which is subject to systemic, correlated shocks. For instance, a symmetric demand shock might resemble the surge in demand for vaccines amid a pandemic, while a symmetric supply shock could be akin to a military conflict causing a spike in energy prices that impacts all manufacturing sectors. Lastly,  $m_k = \frac{1}{n}$ ,  $\forall k$  implies that the sizes of each market segment are equal. The intermediate good producers are uniformly distributed around the unit circle at equidistant intervals.

It is important to note that fully symmetric shocks to final good demand (Q) and intermediate good supply (w) do not immediately imply fully symmetric equilibrium outcomes. For instance, final good producers that are further away from intermediate good supplier nodes (i.e. those with less substitutable inputs) will need to order more of a given input - compared to another final good firm that is

<sup>&</sup>lt;sup>15</sup>Our result may be generalized by assuming either that there is a fixed cost to going to the market or to buying from any specific firm with whom one does not have a previous relation. The problem would become analytically more challenging, but the main insights would stay qualitatively the same

<sup>&</sup>lt;sup>16</sup>This is a slight abuse of notation. We use  $Q_i$  and  $w_j$  to represent both the random variable and its realized value. The intended meaning should be clear within the given context.

closer - to meet the same level of final good demand. In practice, perfectly correlated shocks are the most challenging for resilience, which makes them a key "test case" to examine.

Before diving into the formal equations that define the decentralized equilibrium, it is useful to first explore the more straightforward problems of an unconstrained and constrained social planner. These will serve as our benchmarks for comparison.

### 3 The social planner benchmarks

We characterize the symmetric equilibrium outcomes for two separate benchmarks. In the first, the social planner can perfectly observe the realization of the state variables (Q, w) before committing to intermediate goods production across the network. The planner can therefore perfectly adjust both input factors (L, K) in line with market conditions. We call this the *first-best perfect foresight benchmark*. We re-introduce the informational and technological constraints faced by private agents in the second - *constrained optimal* - social planner's benchmark. Of the two, the constrained optimal benchmark provides a more accurate benchmark. However, the perfect foresight benchmark serves a valuable role in isolating the effects of real-world frictions—such as uncertainties around states and limitations in production technology—from those associated with the distortions that arise due to market externalities and other imperfections.

There are two key distinctions between the social planner (under both benchmarks) and the decentralized market. First, a social planner can directly allocate order flows  $\{q_{ij}\}$  without the need to use price signals  $(\mathbf{p}, \phi)$  as a coordinating mechanism. Second, a social planner maximizes the welfare of the economy as a whole, whereas individual private agents maximize their own profit/utilities. Thus the social planner internalizes any externalities that may arise.

We restrict attention to a "full production equilibrium", where the total demand for final goods can be met in a socially profitable way (i.e. where, at the margin, the value of the final good exceeds the marginal cost of production). This setting further underscores that our core findings are not contingent on the occurrence of rare, large-scale shocks. Formally, a symmetric economy  $\mathscr{E} = \{f(\cdot), \alpha, w, r, Q; v\}$  admits a *full production equilibrium* if there exists an equilibrium whereby  $\tilde{Y}_i(Q, w) = Q$ ,  $\forall i \in [0, 1]$  and for all states of the world  $(Q, w) \subset \mathbb{R}^2_+$ .

**Assumption** A2 [Full production]: We provide conditions on the model primitives which ensure that a symmetric economy can achieve a full production equilibrium. More specifically, we assume that at every point around the circle (i.e.  $\forall i \in I = [0,1]$ ), the marginal benefits of producing final goods will at least match or exceed the marginal costs in all possible scenarios:

$$\frac{v}{f\left(\frac{1}{2n}\right)} \ge \left(\frac{\bar{w}}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha} \left(\frac{\bar{w}\bar{Q}^{\frac{1}{\alpha}}}{E\left[wQ^{\frac{1}{\alpha}}\right]}\right)^{1-\alpha} \quad \text{for } n \ge 2$$
 (3.1)

Assumption A2 states that, the marginal benefit of delivering intermediate goods to the final good producer located furthest to the nearest node (at a distance of  $\frac{1}{2n}$ ) is weakly greater than the marginal cost of producing the intermediate good  $\left(\frac{\bar{w}}{\alpha}\right)^{\alpha}\left(\frac{r}{1-\alpha}\right)^{1-\alpha}\left(\frac{\bar{w}\bar{Q}^{\frac{1}{\alpha}}}{E\left[wQ^{\frac{1}{\alpha}}\right]}\right)^{1-\alpha}$ , even when the negative supply shock is at its most extreme  $(w=\bar{w})$ , and demand is at its upper bound  $(Q=\bar{Q})$ . For any given v, this assumption is equivalent to a restriction on the range of the demand and supply shocks. The assumption guarantees full production under the constrained optimal benchmark, where the social planner faces the same informational and technological constraints as the decentralized market. The corresponding condition for the perfect foresight benchmark is  $\frac{v}{f\left(\frac{1}{2n}\right)} \geq \left(\frac{\bar{w}}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha}$  for  $n \geq 2$ , where the marginal cost of production is lower because the social planner can fully adjust both inputs of production (K as well as L) in response to shocks (i.e.  $\bar{w}\bar{Q}^{\frac{1}{\alpha}} > E\left[wQ^{\frac{1}{\alpha}}\right]$  by construction). Assumption A2 is therefore a sufficient condition for full production under both social planner benchmarks. On a technical note, the full production assumption also enables us to avoid problems of non-differentiability in the demand function.

<sup>&</sup>lt;sup>17</sup>See Appendix C.2 for details.

<sup>&</sup>lt;sup>18</sup>See Appendix D for a more detailed discussion.

Relaxing Assumption A2 leads to cases where some segment of the economy (furthest away from the intermediate good producers) might become shut out from the final good market under adverse supply conditions. In such instances, intermediate goods suppliers operate as localized monopolies rather than as direct competitors, each prioritizing the needs of their local markets (at higher margins) and leaving demand from more "distant" firms unfulfilled. The emergence of local monopolies introduces an extra layer of distortion to the decentralized market solution, which further strengthens our core argument that there is insufficient investment in non-scalable production capacity. <sup>19</sup>

### 3.1 The Perfect Foresight (PF) benchmark

Consider the first-best problem for a social planner with a fully scalable production function and perfect foresight. The social planner operates a standard Cobb-Douglas production function for intermediate goods:  $Y_{j,t} = L_{j,t}^{\alpha} K_{j,t}^{1-\alpha}$ . The planner can also dictate input choices  $\{K_j, L_j\}_{j \in J}$  and order flows  $\{q_{ij}\}_{i \in I, j \in J}$  for all firms after observing the realization of final good demand Q and the scalable input cost w. Although production is delayed until period 1, there is no uncertainty. At period 0, firms know the realization of the shocks that arrive at period 1. Mathematically, this is equivalent to all decisions being made in a single period optimization problem, where the objective is to maximize the value of production net of its costs.

<sup>&</sup>lt;sup>19</sup>We discuss the consequences of relaxing this assumption in greater detail in Appendix G.

 $<sup>^{20}</sup>$ We suppress the t subscript henceforth to simplify notation.

### [Optimization Problem PF]:

$$W(Q, w) = \max_{\{K_j\}_{j \in J}, \{L_j\}_{j \in J}, \{q_{ij}\}_{i \in I, j \in J}} \left\{ v \int_0^1 \left[ \min \left\{ Q, \tilde{Y}_i \right\} \right] di - \sum_{j \in J} \left[ rK_j + wL_j \right] \right\}$$
(3.2)

s.t. 
$$\tilde{Y}_i = \sum_{j \in J} \frac{1}{f_{ij}} q_{ij}$$
 [Production function for final good i] (3.3)

$$Y_j = L_j^{\alpha} K_j^{1-\alpha} \quad \forall j \in J \quad [Production function for intermediate good j]$$
(3.4)

$$\int_{0}^{1} q_{ij} di \leq Y_{j} \quad \forall j \in J \quad [\text{Feasibility of intermediate goods order flow}]$$

$$q_{ij} \geq 0 \quad \forall i \in [0, 1], \forall j \in J \quad [\text{Non-negative inputs}]$$
(3.5)

$$q_{ij} \ge 0 \quad \forall i \in [0,1], \forall j \in J \quad [\text{Non-negative inputs}]$$
 (3.6)

The solution is simple and intuitive. In the perfect foresight benchmark, the planner would meet final good demand by sourcing intermediate good inputs from the cheapest supplier, and produce the required intermediate goods at minimal cost by optimizing the ratio between scalable and non-scalable inputs in every state.

### Proposition 1. [Full production symmetric equilibrium under perfect foresight]:

1. The social planner allocates sufficient intermediate goods j to each final good firm i to meet consumer demand Q, accounting for any imperfect substitutability  $f_{ij}$ . The required intermediate good inputs will be sourced from the lowest effective-cost supplier(s) for each i, whenever the value of production v exceeds the marginal cost of production:

$$q_{ij}^{PF}(Q,w) = \begin{cases} f_{ij}Q & \text{if } j \in \underline{J}(i) \text{ and } v \ge f_{ij} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha} \\ 0 & \text{otherwise} \end{cases}$$
(3.7)

where 
$$\underline{J}(i) := \left\{ \tilde{j} \in J \middle| f_{i\tilde{j}} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \le f_{ij} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \quad \forall j \in J \right\}$$
 is the set of lowest effective-cost supplier(s).

2. The planner's input choices in intermediate good production satisfy the opti-

mality condition:

$$\alpha r K^{PF}(Q, w) = (1 - \alpha) w L^{PF}(Q, w)$$
(3.8)

which yields the explicit solution:

$$K^{PF}(Q, w) = \left(\frac{w}{r} \frac{(1-\alpha)}{\alpha}\right)^{\alpha} \left(2Q \int_{0}^{\frac{1}{2n}} f(i) di\right)$$
(3.9)

$$L^{PF}(Q, w) = \left(\frac{r}{w} \frac{\alpha}{1 - \alpha}\right)^{1 - \alpha} \left(2Q \int_0^{\frac{1}{2n}} f(i) di\right)$$
(3.10)

where  $f(i) = f_{i0} := f(d(i,0))$  is the short-hand for the distance penalty between final good firm i and intermediate good firm 0; and by symmetry  $K_i^{PF} = K^{PF}$  and  $L_i^{PF} = L^{PF}$  for all  $j \in J$ .

### 3.2 The Constrained Optimal Social Planner (SP) Benchmark

Next, we consider the constrained optimal problem, whereby a social planner can dictate production choices  $\{L_j, K_j\}$  and order flow  $\{q_{ij}\}_{i \in I, j \in J}$ ; but is subject to the same informational and technological limitations as the private sector. We solve the constrained optimal problem through backward induction.

In period 1, the social planner takes the pre-committed non-scalable capacity  $K_j = K$ ,  $\forall j \in J$  as given, and chooses the scalable input factor  $\left\{L_j\right\}_{j \in J}$  and order flows  $\left\{q_{ij}\right\}_{i \in I, j \in J}$  to maximize aggregate welfare for given realization of demand and supply conditions (Q, w). For given intermediate good output  $Y_j = \int_0^1 q_{ij}di$ , we can express the cost-minimizing level of the scalable factor as  $L_j = \left(\int_0^1 q_{ij}di\right)^{\frac{1}{\alpha_j}}K_j^{-\frac{\left(1-\alpha_j\right)}{\alpha_j}}$ . Substituting out  $L_j$ , and imposing symmetry (Assumption A1), we can express the optimization problem [SP1] in terms of the order flows  $\left\{q_{ij}\right\}_{i \in I, j \in J}$  only:

[Optimization problem SP1]:

$$W^{SP}(K|Q,w) = \max_{\left\{q_{ij}\right\}_{i\in I,j\in J}} \left\{ v \int_{0}^{1} \left( \sum_{j\in J} \frac{1}{f_{ij}} q_{ij} \right) di - \sum_{j\in J} \left[ rK + w \left( \left( \int_{0}^{1} q_{ij} di \right)^{\frac{1}{\alpha}} K^{-\frac{(1-\alpha)}{\alpha}} \right) \right] \right\}$$

$$(3.11)$$

s.t. 
$$Q \ge \sum_{i \in I} \frac{1}{f_{ij}} q_{ij} \quad \forall i \in [0, 1] \quad [Demand cap]$$
 (3.12)

$$q_{ij} \ge 0 \quad \forall i \in [0,1], j \in J \quad [\text{Non-negative inputs}]$$
 (3.13)

where  $v \int_0^1 \left( \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} \right) di$  is the aggregate value derived from the production of final goods, and  $\sum_{j \in J} \left[ rK + w \left( \left( \int_0^1 q_{ij} di \right)^{\frac{1}{\alpha}} K^{-\frac{(1-\alpha)}{\alpha}} \right) \right]$  the aggregate cost of producing the necessary intermediate inputs. The demand cap reflects that any production in excess of the realized demand Q will be wasted.

Back in period 0, the social planner chooses non-scalable inputs  $\{K_j\}_{j\in J}$  to maximize expected welfare in period 1, accounting for the probability distribution of demand and supply shocks (Q, w).

### [Optimization problem SP0]:

$$W^{SP} = \max_{K} E\left[W^{SP}(K|Q, w)\right]$$

The solution resembles that of the perfect foresight scenario, but with important distinctions, arising from the necessity of committing to a specific level of capacity investment in period 0, prior to the realization of states in period 1.

## Proposition 2. [Full production symmetric equilibrium in the constrained optimal benchmark]

1. In period 1, the social planner allocates sufficient intermediate goods to each final good firm i to meet consumer demand Q, accounting for imperfect substitutability. The required intermediate good inputs will be sourced from the lowest effective-cost supplier(s) for each i, whenever the value of production

<sup>&</sup>lt;sup>21</sup>In this analysis, we deliberately exclude the impact of inventory management due to the framework's static, one-shot nature. See Ferrari (2022) for a network model with inventories.

v exceeds the marginal cost of production:

$$q_{ij}^{SP}(Q,w) = \begin{cases} f_{ij}Q & \text{if } j \in \underline{J}(i) \text{ and } v \ge f_{ij}\widetilde{MC} \\ 0 & \text{otherwise} \end{cases}$$
(3.14)

where  $\widetilde{MC} := \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha} \left(\frac{wQ^{\frac{1}{\alpha}}}{E\left[wQ^{\frac{1}{\alpha}}\right]}\right)^{1-\alpha}$  is the marginal cost of producing the intermediate good in the symmetric equilibrium; and  $\underline{J}(i) := \left\{\widetilde{j} \in J \middle| f_{i\widetilde{j}}\widetilde{MC} \leq f_{ij}\widetilde{MC} \quad \forall j \in J\right\}$  is the set of lowest effective-cost supplier(s). The optimal level of scalable input is given by:

$$L^{SP}(Q,w) = \left(\int_0^1 q_{ij}^{SP}(Q,w) di\right)^{\frac{1}{\alpha}} \left(K^{SP}\right)^{-\frac{(1-\alpha)}{\alpha}}$$
(3.15)

2. In period 0, the optimal level of non-scalable production capacity  $K^{SP}$  satisfies the optimality condition:

$$\alpha r K^{SP} = (1 - \alpha) E \left[ w L^{SP} \right]$$
 (3.16)

which can be solved explicitly to give:

$$K^{SP} = \left(\frac{1}{r}\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(2\int_{0}^{\frac{1}{2n}} f(i) \, di\right) \left(E\left[wQ^{\frac{1}{\alpha}}\right]\right)^{\alpha} \tag{3.17}$$

3. The relationship between capacity investment across the two benchmark scenarios can be summarized as follows:

$$K^{SP} = K^{PF}(Q, w) \left( \frac{E\left[ wQ^{\frac{1}{\alpha}} \right]}{wQ^{\frac{1}{\alpha}}} \right)^{\alpha}$$
 (3.18)

and by Jensen's inequality we have:

$$K^{SP} \ge E\left[K^{PF}\left(Q,w\right)\right] \tag{3.19}$$

The first part of the proposition relating to the optimal order flow  $(q_{ij}^{SP})$  and the level of scalable capacity  $(L^{SP})$  is straight-forward. Here we will focus discussions on the intuition behind the constrained optimal solution for capacity investment  $K^{SP}$ . The choice of non-scalable capacity at period 0,  $K^{SP}$ , influences aggregate welfare in period 1 through two primary mechanisms. First, any increase in  $K^{SP}$ generates a *direct cost* given by r. This cost, however, is partly offset by the resultant decrease in the scalable input  $L^{SP}$  needed to achieve a given output Y thus offering a direct benefit. Second, an increase in capacity  $K^{SP}$  may increase aggregate intermediate good production Y, and indirectly improve welfare through this output channel  $\frac{dY}{dK}$ . However, in a full-production equilibrium where the demand for final goods is always met (i.e., the demand cap is binding), there can be no further welfare gains from increasing aggregate intermediate good production. Therefore, the indirect impact of K on welfare is exactly zero.<sup>22</sup> This leaves us with the familiar optimality condition that is typical for Cobb-Douglas production functions,  $\alpha r K^{SP} = (1 - \alpha) E \left[ w L^{SP} \right]$ , albeit with an expectation function to account for the ex-ante uncertainty.

Finally, equation 3.18 illustrates the relationship between the level of capacity investment across the two benchmarks. Under the constrained optimal benchmark, the social planner must commit to a given level of capacity  $K^{SP}$  before observing the shocks. Hence, capacity investment is lower than the perfect foresight case,  $K^{SP} < K^{PF}(Q,w)$ , in states where marginal costs exceed expectations (when w and/or Q are higher than expected). Conversely,  $K^{SP} > K^{PF}(Q,w)$  when marginal costs fall below expectations. Importantly, this implies that the constrained social planner recognizes that investing in a level of production capacity that accommodates all contingencies  $(K^{PF}(\bar{Q},\bar{w}))$  would give rise to a supply network which is inefficiently resilient. Nevertheless, the constrained social planner in-

<sup>&</sup>lt;sup>22</sup>In the formal proof (see appendix C.2), we show that the optimality condition for non-scalable production capacity  $K^{SP}$  (equation 3.16) remains unchanged when we relax the full production assumption. We can safely ignore the indirect effects of K on welfare through changes in output Y, because these indirect effects are multiplied by the difference between the marginal cost and marginal benefit of production for the threshold buyer, which is equal to zero by construction. This result bears resemblance to the Envelope Theorem, in which the total derivative of the value function with respect to the parameters of the model is equal to its partial derivative. Here K is the choice variable, but the total derivative of  $W^{SP}(K|Q,w)$  with respect to K is also equal to its partial derivative.

vests in more capacity than its counterpart with perfect foresight does on average,  $K^{SP} \ge E\left[K^{PF}\left(Q,w\right)\right]$ , as a way to insure against uncertainty. A formal exposition of these results can be found in appendix C.3.

# 4 The decentralized solution: equilibrium in the spot and pre-order markets

In the decentralized market equilibrium, firms adjust production in response to prices in both the pre-order and spot market. We solve the model through backward induction.

### 4.1 Period 1 equilibrium in the spot market

In period 1, each final goods producer can turn to the spot market to acquire additional intermediate goods beyond those which have been pre-ordered. Formally, each final good producer i takes realized demand for final goods  $Q_i$ , prior commitments  $\mathbf{q}_i^{pre}$ , pre-order and spot intermediate good prices  $(\phi, \mathbf{p})$  as given<sup>23</sup>; and purchases intermediate goods  $\mathbf{q}_i^{spot}$  from intermediate good producers on the spot market in order to maximize profit:

$$\Pi_{i}\left(\mathbf{q}_{i}^{pre}, \phi, \mathbf{p}\right) = \max_{\mathbf{q}_{i}^{spot}} \left\{ v \min\left\{Q_{i}, \tilde{Y}_{i}\right\} - \tilde{C}_{i}\left(\mathbf{q}_{i}^{pre}, \mathbf{q}_{i}, \phi, \mathbf{p}\right) \right\}$$
(4.1)

s.t. 
$$\mathbf{q}_i^{spot} \ge 0$$
 [No-default constraint] (4.2)

where final good production and total costs are given by:

$$\tilde{Y}_i = \sum_{j \in J} \frac{1}{f_{ij}} \left( q_{ij}^{spot} + q_{ij}^{pre} \right) \tag{4.3}$$

$$\tilde{C}_{i}\left(\mathbf{q}_{i}^{pre}, \mathbf{q}_{i}, \phi, \mathbf{p}\right) = \phi \cdot \mathbf{q}_{i}^{pre} + \mathbf{p} \cdot \mathbf{q}_{i}^{spot}$$

$$(4.4)$$

<sup>&</sup>lt;sup>23</sup>As is conventional in the literature on Bertrand equilibria, each firm assumes he can buy as much on the spot market as he wishes. This assumption is particularly important for the analysis of firm's decision-making at time 0.

We interpret  $\mathbf{q}_i^{spot} \geq 0$  as a "no-default constraint" because it implies that the total volume of intermediate good orders will never fall below the pre-ordered amount:  $\mathbf{q}_i := \mathbf{q}_i^{spot} + \mathbf{q}_i^{pre} \geq \mathbf{q}_i^{pre}$ . The final good producers cannot renege on the promises made in period 0. In principle, a firm could also resell his pre-order to some other firm, so that the level of input could be less than the pre-ordered level. But in a symmetric equilibrium that never occurs.<sup>24</sup>

If the spot market were perfectly competitive, each intermediate supplier would produce up to the point where the price of the intermediate good (on the spot market) were equal to the *marginal* cost of production, and the demand for intermediate goods would be determined in the usual way, with equilibrium in the spot market occurring at the price where demand equals supply. But this is instead a highly differentiated market for intermediate goods, and each intermediate good producer acts in a monopolistically competitive way, setting a spot price  $p_j$ , taking its non-scalable production capacity  $K_j$ , and the price of its competitors  $\mathbf{p}_{-j}$  as given. Preorder contracts  $\left\{q_{ij}^{pre}\right\}_{i\in I}$  are honored at the agreed price  $\phi_j$ . The profit of firm j is given by its pre-order revenue plus spot market revenue, minus the total costs of production:

$$\Pi_{j}\left(K_{j},\left(\phi_{j},q_{j}^{pre}\right),\mathbf{p}_{-j}\right) = \max_{p_{j}}\left\{\left[\phi_{j}Y_{j}^{pre}\right] + \left[p_{j}Y_{j}^{spot}\right] - \left[w_{j}L_{j}^{*} + r_{j}K_{j}\right]\right\} \quad (4.5)$$

where

$$Y_j^{pre} := \int_0^1 q_{ij}^{pre} di \tag{4.6}$$

$$Y_j^{spot} := \int_0^1 q_{ij}^{spot} di \tag{4.7}$$

are the level of intermediate good production required to meet pre-order demands and spot market demands respectively.

<sup>&</sup>lt;sup>24</sup>Conceptually, we could imagine an equilibrium where say in some states, those in one set of locations sold excess pre-orders to those in another set of locations. Our assumption of perfectly correlated shocks is what rules this out. Alternatively, even with imperfectly correlated shocks, reselling excess orders can be assumed away, e.g., because there are some (not fully specified here) adaptations of production to each producer, which make such sales impossible. In practice resale of pre-ordered inputs do occur, though they are likely limited in scale.

Similar to our treatment of the social planner benchmarks, we restrict attention to a full production symmetric equilibrium for analytical tractability. In a symmetric setting all intermediate good firms  $j \in J$  share the same characteristics  $\alpha_j = \alpha$ ,  $r_j = r$  and  $w_j = w$ ; and every final good firm  $i \in [0,1]$  will face the same exogenous demand  $Q_i = Q$ . In equilibrium, input choices will be the same across intermediate good firms:  $K_j = K$  and  $L_j = L$ ,  $\forall j \in J$ ; and final good firms will fulfill the same proportion of the realized demand for final goods through the pre-order market:  $Q_i^{pre} := \sum_j \frac{1}{f_{ij}} q_{ij}^{pre} = Q^{pre} \ \forall i \in [0,1]$ .

It is important to note that  $Q_i^{pre}$  denotes the level of *final good* demand that is fulfilled through pre-orders, and not the quantity of *intermediate goods* pre-ordered  $\mathbf{q}_i^{pre}$ . The link between the two is given by  $Q_i^{pre} := \sum_j \frac{1}{f_{ij}} q_{ij}^{pre}$ , where  $\frac{1}{f_{ij}}$  accounts for imperfect substitutability. Later, in Section 4.2, we show that  $Q_i^{pre} = Q^{pre} \ \forall i \in [0,1]$  is indeed an optimal equilibrium strategy in period 0, but this implies pre-orders for intermediate goods  $q_{ij}^{pre}$  are not be equalized across i's.

**Proposition 3.** [Full Production Symmetric Equilibrium in the spot market] In period 1, taking period 0 choices  $(\{\mathbf{q}_i^{pre,*}\}, K^*, \phi^*)$  as given:

1. Final good firms order intermediate goods on the spot market from the supplier offering the lowest effective-prices  $j \in \underline{J}(i; \mathbf{p}) := \left\{ \tilde{j} \in J : f_{i\tilde{j}} p_{\tilde{j}} = \min \left\{ \mathbf{f}_i \circ \mathbf{p} \right\} \right\}$ :

$$q_{ij}^{spot,*} = \begin{cases} f_{ij} \left( Q - Q_i^{pre} \right) & \text{if } Q \ge Q_i^{pre}, j \in \underline{J}(i; \mathbf{p}), \text{ and } v \ge f_{ij} p_j \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in [0, 1]$$

$$(4.8)$$

- 2. Intermediate good firms:
  - purchase the cost minimizing level of scalable inputs:

$$L_{i}^{*} = L^{*} = (Y^{pre,*} + Y^{spot,*})^{\frac{1}{\alpha}} (K^{*})^{-\frac{1-\alpha}{\alpha}} \quad \forall j \in J$$
 (4.9)

• set spot-market prices at a mark-up over marginal costs:

$$p_j^* = p^* = \underbrace{(1+\mu)}_{mark-up>1} MC \quad \forall j \in J$$
 (4.10)

where

- $Y^{pre,*} := \int_0^1 q_{ij}^{pre,*} di$  and  $Y^{spot,*} := \int_0^1 q_{ij}^{spot,*} di$  are the level of intermediate good production required to meet equilibrium pre-order demand and spot market demand respectively;
- $\mu$  is the proportional mark-up over marginal costs given by:

$$\mu := \frac{2f'\left(\frac{1}{2n}\right) \int_0^{\frac{1}{2n}} f(i) di}{\left(f\left(\frac{1}{2n}\right)\right)^2 - 2f'\left(\frac{1}{2n}\right) \int_0^{\frac{1}{2n}} f(i) di}$$
(4.11)

- MC is the marginal cost faced by intermediate good suppliers:

$$MC = \frac{w}{\alpha} \left( \frac{Y^{pre,*} + Y^{spot,*}}{K^*} \right)^{\frac{1-\alpha}{\alpha}}$$
(4.12)

In the period 1 equilibrium, each final good producer first evaluates whether their pre-committed orders for intermediate goods will be adequate to satisfy the existing demand for final goods (i.e., whether  $Q_i^{pre} := \sum_j \frac{1}{f_{ij}} q_{ij}^{pre} \ge Q_i$ ). Should the pre-orders prove sufficient, the final good producer i will eschew the spot market, setting  $\mathbf{q}_i^{spot} = 0$ . Otherwise, additional intermediate goods will be purchased on the spot market to meet realized demand, provided that the cost of doing so is less than the value of the output v. Spot market purchases are made from the cheapest intermediate good producer, adjusting for the distance-based penalties (eqn 4.8).

For intermediate good producers,  $L^*$  is the cost-minimizing choice for given capacity investment  $K^*$  (eqn 4.9). Equation 4.10 characterizes the optimal spot market pricing. Intermediate good producers engage in monopolistic competition and charge a mark-up over marginal costs. This mark-up is higher when substitutability is poor for the marginal buyer (i.e. when  $f'(\frac{1}{2n})$  is high); and lower when competition is fierce (i.e. when n is large). In the limit, as n approaches infinity - such that the distance between nodes shrinks to zero and intermediate goods become perfect substitutes - equation 4.10 simplifies down to price equals marginal cost (perfect competition). We explicitly assume that intermediate goods producers cannot engage in price discrimination, charging those at a greater distance less than those nearby. This is a natural assumption in this context: intermediate goods pro-

ducers may not fully observe the characteristics of the firms who seek to buy from them.

From equations 4.10 and 4.12 we see that non-scalable capacity K plays a key role through the marginal cost function. Higher capacity investments by any firm j in period 0 reduces its marginal cost of production in every state in period 1 (though more so in some states than others). However, this decrease in marginal cost does not directly translate into proportionate increases in profit, especially if competing firms also expand their capacities, which would drive down the equilibrium spot price and pass on gains to final good producers. This has important implications for investment in capacity, as the next section shows.

### 4.2 Period 0 equilibrium in the pre-order market

In period 0, the final good producers take pre-order prices  $\phi$  as given, form expectations over the state contingent distribution of spot prices at period 1, and submit pre-orders for intermediate goods  $\mathbf{q}_i^{pre}$  to maximize their expected profit:

$$\max_{\mathbf{q}_{i}^{pre}} E\left[\Pi_{i}\left(\mathbf{q}_{i}^{pre}; \phi, \mathbf{p}^{*}\right)\right]$$

$$=vE\left[Q\right] - \Pr\left(Q > Q_{i}^{pre}\right) E\left[\mathbf{p}^{*}\left(Q, w\right) \cdot \mathbf{q}_{i}^{spot, *}\left(Q, w\right) | Q > Q_{i}^{pre}\right] - \phi \cdot \mathbf{q}_{i}^{pre} \quad (4.13)$$

where  $\Pi_i$  is the profit of firm i in period 1 (eqn. 4.1);  $\mathbf{q}_i^{pre} \coloneqq \left(q_{i0}^{pre}, q_{i1}^{pre}, \ldots, q_{i,n-1}^{pre}\right)'$  is the vector of pre-orders for intermediate goods;  $\phi \coloneqq \left(\phi_0, \phi_1, \ldots, \phi_{n-1}\right)'$  the menu of pre-order prices, and  $Q_i^{pre} \coloneqq \sum_j \frac{1}{f_{ij}} q_{ij}^{pre}$  the volume of final good demand that can be met given the pre-orders and the linear production function for final goods. The final good producer anticipates that the realized demand for final goods Q may fall short of what could be produced from pre-orders  $Q_i^{pre}$  with probability  $\left(1 - \Pr\left(Q > Q_i^{pre}\right)\right)$ . In such a scenario, the final good producer will eschew the spot market in period 1, and not incur any additional costs beyond those associated with the pre-orders. With complement probability  $\Pr\left(Q > Q_i^{pre}\right)$ , the final good producer will need to purchase additional intermediate inputs on the spot market, at

<sup>&</sup>lt;sup>25</sup>As discussed in the previous section, we have imposed a constraint  $\mathbf{q}_i^{spot} \geq 0$  ruling out the resale of pre-ordered intermediate goods.

expected cost 
$$E\left[\mathbf{p}^{*}\left(Q,w\right)\cdot\mathbf{q}_{i}^{spot,*}\left(Q,w\right)|Q>Q_{i}^{pre}\right].$$

Simultaneously, each intermediate good producer j sets pre-order price  $\phi_j$  taking its competitors' prices  $\phi_{-j}$  as given; and commits to a level of non-scalable input factor  $K_j$  in order to maximize expected profit in period 1,

$$\max_{K_{j},\phi_{j}} E\left[\Pi_{j}\left(K_{j},\left\{\phi_{j},\phi_{-j}\right\},\mathbf{q}_{j}^{pre,*}\right)\right] = \left[\phi_{j}Y_{j}^{pre,*} - rK_{j}\right] + E\left[p_{j}^{*}Y_{j}^{spot,*} - wL_{j}^{*}\right]$$

$$(4.14)$$

where  $Y_j^{pre,*}$  and  $Y_j^{spot,*}$  are the intermediate good output required to meet equilibrium pre-orders and spot-market orders respectively (defined in eqns 4.6, 4.7).

We show that in a full production symmetric equilibrium, the optimal pre-order price is equal to the unconditional expectation of spot market prices. Without a discount over expected spot market prices, final good firms pre-order only what is necessary to cover the lowest realization of demand. The restrained demand for pre-orders affects the intermediate good producer's incentive to invest in non-scalable production capacity.

### **Proposition 4.** [Full Production Symmetric Equilibrium in the pre-order market] In period 0:

1. Each final good producer i pre-orders only what is necessary to cover the lowest realization of final good demand from its nearest intermediate good supplier:

$$q_{ij}^{pre,*} = \begin{cases} f_{ij}\underline{Q} & \text{if } j \in \underline{J}(i;\phi), \text{ and } v \ge f_{ij}\phi_j \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in [0,1]$$
 (4.15)

where  $\underline{J}(i;\phi) \coloneqq \left\{ \tilde{j} \in J : f_{i\tilde{j}}\phi_{\tilde{j}} = \min\{f_i \circ \phi\} \right\}$  denote the set of suppliers that provides the lowest effective pre-order price for i (which is equivalent under symmetry to the set of the nearest suppliers).

- 2. Each intermediate good producer j:
  - (a) sets pre-order prices to the unconditional expectation of spot-market

prices

$$\phi^* = E[p^*(Q, w)] \tag{4.16}$$

(b) invests in a level of non-scalable capacity  $K^*$  given by the optimality condition:

$$\alpha r K^* = (1 - \alpha) E[wL^*] + E\left[\underbrace{\frac{\partial p^*}{\partial K} Y^{spot}}_{<0}\right]$$
(4.17)

There is an important intermediate step to show why final good firms find it optimal in equilibrium to pre-order only what is sufficient to meet the lowest realization of final good demand. In Appendix F (Lemma 2) we characterize final good firms' demand for pre-orders  $(Q_i^{pre,*} := \sum_j \frac{1}{f_{ij}} q_{ij}^{pre,*})$  in terms of the equation:

$$\phi = \Pr\left(Q > Q_i^{pre,*}\right) E\left[p^*\left(Q,w\right) | Q > Q_i^{pre,*}\right] \tag{4.18}$$

where  $\{Q>Q_i^{pre,*}\}$  is the set of states in which the final good firms need to purchase additional intermediate goods from the spot market in period 1. For every (effective) unit of intermediate goods that is pre-ordered in period 0, the final good firm will need to order one fewer unit on the spot market - but only in states where  $Q>Q_i^{pre,*}$ . Thus, for a given pre-order price  $\phi$ , final good firms will pre-order just enough intermediate goods such that the  $\phi$  is equal to the expected marginal savings on the spot market, accounting for the fact that larger pre-orders reduces the probability that spot market purchases will be required.

The demand function for pre-orders (characterized by eqn 4.18) has two immediate implications: (1) aggregate pre-orders must be equalized across i in equilibrium ( $Q_i^{pre,*} = Q^{pre,*}$  for all i); and (2) the maximum sustainable pre-order price is the unconditional expectation of the spot market price  $\phi = E[p^*]$ . Since final good firms are risk neutral, they will not pre-order if  $\phi > E[p^*]$ . Likewise, intermediate good firms do not have incentives to offer a discount on pre-orders (i.e. pay a premium for insurance) by setting  $\phi < E[p^*]$ . Intermediate good producers do not have incentive to reduce  $\phi$  below  $E[p^*]$  to attract more pre-orders because they expect to

make more marginal profit on the spot market. Critically, any extra marginal costs incurred from lower capacity investments can also be passed on to final good firms on the spot market along with a mark-up. In fact, because the spot market mark-up is proportional to marginal costs, and aggregate output remain unchanged in a full production equilibrium, final good firms' profits measured in dollar terms is actually higher when there are symmetric and correlated negative supply shocks. With market power on the spot market, intermediate good firms see no need to promote pre-orders to insure against correlated adverse supply shocks.

In equilibrium, therefore, we have a corner solution with  $\phi = E[p^*]$ , and  $Q^{pre,*} = \underline{Q}$ . Intermediate good firms set pre-order prices at the level that makes final good firms indifferent between no pre-orders at all and pre-ordering only what is necessary to cover the lowest realization of demand  $\underline{Q}$ . In short, intermediate good firms sets the highest possible pre-order price that drives the final good firms to their participation constraint.<sup>26</sup>

Having characterized the equilibrium quantity and price of pre-orders, the intermediate good suppliers determine the amount of production required to meet pre-orders  $(Y^{pre,*})$ ; and forecast expected prices  $(p^*)$  and production on the spot market  $(Y^{spot,*})$ . The intermediate good suppliers then invest in a level of non-scalable capacity  $K^*$  that minimizes expected costs for the anticipated level of production (equation 4.17). This optimality condition for  $K^*$  is similar to its analogues under the social planner benchmarks (equations 3.8 and 3.16 for the unconstrained and constrained cases, respectively); except for the addition of a final term  $E\left[\frac{\partial p^*}{\partial K}Y^{spot}\right]$ . This final term captures the pecuniary externality that arise from enhanced market power, and the over-reliance on spot markets. It plays an important role in explaining the wedge between decentralized market solution and the constrained optimal benchmark.

 $<sup>^{26}</sup>$ Both the full production and the symmetry assumption play an important role here. We no longer have  $\phi = E\left[p^*\right]$  as an equilibrium condition when these assumptions are relaxed. Likewise, we will also move away from this corner solution if agents are risk-averse, though the presence and qualitative properties of the market failures we identify are likely to be same. That is, while with risk aversion there is likely to be more investment in capacity (greater resilience) in the market equilibrium, with more risk averse agents, (constrained) Pareto optimality also requires greater resilience, and there will remain a gap between the two.

## 5 Decentralized solution vs constrained optimal benchmark

We can now prove the core proposition of the paper. The level of investment in the non-scalable capacity in a decentralized market setting  $(K^*)$  is suboptimally low when compared to the level in the constrained optimal benchmark  $(K^{SP})$ .

### **Proposition 5.** [Sub-optimal non-scalable capacity investment] $K^* < K^{SP}$ .

*Proof.* We prove  $K^* < K^{SP}$  by contradiction. This proof is instructive because it highlights the importance of the pecuniary externality  $\frac{dp^*}{dK}$  and the over-reliance on the spot market  $Y^{spot}$  as the main drivers behind the under-investment in capacity.

First, by the full-production assumption we know that the level of intermediate good production is the same under both the decentralized solution and the constrained benchmark:  $Y^*(Q, w) = Y^{SP}(Q, w)$ , in all states of the world (Q, w).

This implies that if  $K^* = K^{SP}$ , then  $L^*(Q, w) = L^{SP}(Q, w)$  in every state, leading to a contradiction:

$$\alpha r K^{SP} = (1 - \alpha) E \left[ w L^{SP} \right] = (1 - \alpha) E \left[ w L^* \right]$$

$$> (1 - \alpha) E \left[ w L^* \right] + E \left[ \underbrace{\frac{dp^*}{dK} Y^{spot}}_{<0} \right] = ar K^*$$

If instead  $K^* > K^{SP}$ , then  $L^*(Q, w) < L^{SP}(Q, w)$  in every state, again giving rise to a contradiction:

$$\alpha r K^* = E \left[ \underbrace{\frac{dp^*}{dK^*} Y^{spot}}_{<0} \right] + (1 - \alpha) E \left[ wL^* \right]$$

$$< (1 - \alpha) E \left[ wL^* \right] < (1 - \alpha) E \left[ wL^{SP} \right] = \alpha r K^{SP}$$

The proposition reveals that intermediate goods producers under-invest in capacity upfront because they are unable to fully capture the cost savings generated by increased investment. Specifically, each dollar saved through efficiency gains from capacity investment does not yield a corresponding one-dollar increase in profits. This is because a part of these gains is transferred to final goods producers through lower spot market prices. The key term of interest is  $E\left[\frac{dp^*}{dK}Y^{spot}\right]$ , which captures the interaction between the pecuniary externality  $(\frac{dp^*}{dK}$  the sensitivity of spot market prices to capacity investment) and the degree of reliance on the spot market  $(Y^{spot})$ .

Focusing first on the price sensitivity term  $\frac{dp^*}{dK}$ , recall that equilibrium spot prices can be expressed as a proportional mark-up over marginal costs:  $p^* = (1 + \mu)MC$ . All else equal, higher capacity investment K, lowers the marginal cost ( $MC = \frac{w}{\alpha} \left(\frac{Y^*}{K^*}\right)^{\frac{1-\alpha}{\alpha}}$ ) in every possible state and thus lowers spot prices. The extent to which K matters depends on the *scalability* of the economy ( $\alpha$ ). As scalability improves and  $\alpha \to 1$ , the less important is K in production, and the externality shrinks.

The impact of K on marginal costs is amplified by the mark-up  $(\mu = \frac{2f'\left(\frac{1}{2n}\right)\int_0^{\frac{1}{2n}}f(i)di}{f\left(\frac{1}{2n}\right)-2f'\left(\frac{1}{2n}\right)\int_0^{\frac{1}{2n}}f(i)di})$ .

The size of the mark-up dependends on the *substitutability* between sectors, as measured by the distanced-based penalty function f (evaluated at the marginal buyer  $i = \frac{1}{2n}$ ). Higher substitutability between sectors lowers mark-up and reduces the wedge between the decentralized solution and the constrained optimal benchmark in equilibrium. Lastly, another important way to reduce the wedge is through enhanced competition (i.e. a larger n), which also reduces the amplification of marginal cost changes by reducing mark-ups.

Equally as important, the wedge results from an over-reliance on the spot market. Unlike an Arrow-Debreu economy, where agents can trade contingent claims for every conceivable state of the world, in our model - much like real-world conditions - the set of contracts that can feasibly be written and traded is much smaller than the set of possible states. As a result, the pre-order, forwards, and futures markets will fall short of providing adequate risk insurance for intermediate goods producers. Downstream final goods producers fail to sufficiently compensate their suppliers for the pecuniary externality arising from the benefits of increased capital

### **5.1** Policy Interventions

Using our model, it is possible to identify a number of ways to narrow the wedge between the supply network delivered by unfettered markets and the efficiently resilient network characterized under a constrained optimal benchmark.

The most straightforward strategy to address the externality in the model is to offer subsidies for capacity investments, thereby lowering the effective cost r incurred by intermediate goods producers for non-scalable capacity. Second, the government might extend tax benefits to downstream firms that engage in pre-orders or transact in the futures market, or alternatively, levy additional taxes on spot market transactions. Futures markets facilitate greater risk sharing between upstream and downstream entities, and diminish dependency on spot markets. A third avenue is to reduce the sensitivity of spot prices to changes in capacity investments. This could entail structural economic reforms such as lowering entry barriers (including trade barriers), stronger competition policies, and enhancing the substitutability of intermediate products, all of which could reduce supplier mark-ups. Similarly, technological advancements in production scalability could shift the focus towards other input factors that can be more readily adjusted on short notice.

In practice, it may be harder to device practical, implementable interventions. Directly subsidizing capacity investments offers a straightforward strategy, yet distinguishing such investments from other types of capital expenditure can be difficult, particularly in certain sectors. The government may want to intervene only in certain critical industries - e.g. computer chip production, where downstream externalities are especially significant and resilience is more important - by for instance, offering a lower taxes for firms operating with excess capacity. While tax incentives for spot and pre-order markets can be effective in sectors like electricity, with its well-defined spot and futures markets, this approach becomes less straightforward in industries where market boundaries are more blurred.

<sup>&</sup>lt;sup>27</sup>In a sense, this pecuniary externality is a special case of the general pecuniary externality arising in economies without a complete set of AD securities analyzed by Greenwald and Stiglitz (1986) and first discussed in Stiglitz (1982).

### **6 Concluding Remarks**

Especially since the pandemic and post-pandemic supply chain interruptions, the question of resilience has moved to the fore. Of course, we don't expect markets to be prepared for every shock, regardless of size: doing so would be extraordinarily expensive. The question is, do they make appropriate preparations, measured against an appropriate benchmark? There are many reasons to think that they might not: critics of the market, for instance, complain about short-termism.

We examine the normative question of resilience, however, in a world with fully rational expectations and in which firms do not suffer from short-termism, showing that nonetheless, there is a bias towards excessive vulnerability due to insufficient ex-ante capacity investments by upstream intermediate goods producers. This shortfall arises because these producers cannot fully capture the returns on their capacity investments: a portion of the economic gains is transferred downstream to final goods producers through reduced spot market prices.

We believe that our study is the first to incorporate interactions in both the spot and futures market in such a normative analysis of supply networks, essential for addressing the question at hand. Doing this in the context of differentiated competition necessarily entails a certain degree of complexity. For tractability and ease of exposition, we have introduced a number of simplifications; but in the online appendix G, we show how the results hold under significantly more general conditions. Most notably, we show that if there are very large shocks, such that the cost of meeting the market demand is so high that there are "unserved" customers (i.e. Assumption A2 *Full Production* is not satisfied), then the analysis still holds.

Finally, we note that in certain industries there may be forces pushing in the other direction: some firms may choose to hold excess production capacity purely as a way to deter prospective entrants, thereby reducing competition. Risk aversion on the part of intermediate and final goods producers (and consumers, translated into more profitable contracts signed with firms that have greater resilience) may also result in greater resilience than suggested by this model. Moreover, we have assumed that market power resides in the upstream firms. Especially in more oligopolistic contexts, downstream firms may engage in supply chain diversification

and higher levels of pre-ordering, generating higher levels of capital investment in the upstream industries and greater market resilience, explicitly to limit the ability of the upstream firms to exercise market power in the manner illustrated here. The one result that we believe is resilient is that there is likely to be a disparity between the market and the constrained optimal level of resilience.

The events of the last few years has made it clear that economists have paid insufficient attention to resilience. This paper is intended as a contribution to the nascent literature attempting to understand better why markets may have underinvested in resilience.

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## For Online Publication: Appendix

## **A** Cost functions for Cobb-Douglas production function

#### A.1 Standard Cobb-Douglas Production

The standard cost minimization problem with a Cobb-Douglas production function is given by:

$$\min_{L,K} C = wL - rK$$
s.t.  $L^{\alpha}K^{1-\alpha} \ge Y$ 
(A.1)

Setting up the Lagrangian and computing the necessary first order conditions yields the familiar optimality condition:

$$(1 - \alpha) wL = \alpha rK \tag{A.2}$$

Substituting the optimality condition into the production function yields the optimal input choices  $K = Y\left(\frac{w}{r}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$ , and  $L = Y\left(\frac{r}{w}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$ . The cost function is therefore given by:

$$C(Y) = Y\left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha} \tag{A.3}$$

with constant marginal cost:

$$MC := C'(Y) = \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha}$$
 (A.4)

### A.2 Cobb-Douglas Production with Partial Delay

With partial delay, the intermediate good producer takes K as given in its period 1 cost minimization problem:

$$\min_{L} \tilde{C}(K) = wL + rK$$
s.t. $L^{\alpha}K^{1-\alpha} \ge Y$ 

The optimal L is given simply by the minimum amount necessary to produce Y:

$$L = Y^{\frac{1}{\alpha}} K^{-\frac{(1-\alpha)}{\alpha}} \tag{A.5}$$

The cost function therefore depends on both the desired output Y and the capacity K reserved ex ante:

$$\tilde{C}(Y;K) = wY^{\frac{1}{\alpha}}K^{-\frac{(1-\alpha)}{\alpha}} + rK \tag{A.6}$$

with a marginal cost of production that depends on the output-capacity ratio  $(\frac{Y}{K})$ :

$$\widetilde{MC} := \frac{d\widetilde{C}(Y;K)}{dY} = \frac{w}{\alpha} \left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}$$
 (A.7)

Note that the impact of capacity on total cost is given by:

$$\frac{d\tilde{C}(Y;K)}{dK} = -w \frac{(1-\alpha)}{\alpha} \left(\frac{Y}{K}\right)^{\frac{1}{\alpha}} + r$$

$$= r - (1-\alpha) \left(\frac{Y}{K}\right) \widetilde{MC} \tag{A.8}$$

which is lower than r, because of the additional indirect cost savings on scalable input capacity.

## B Proof for Proposition 1 - Perfect Foresight benchmark

We start with the optimization problem for the perfect foresight benchmark [PF] characterized in the main text (eqns 3.2 to 3.6). With perfect foresight, both L and K can be set as a function of the realized state (Q, w) in period 1. This is equivalent to saying that both L and K can be adjusted flexibly and simultaneously as the need arise. We thus have a standard Cobb-Douglas production for intermediate goods, with: optimal input choices characterized by  $(1-\alpha)wL = \alpha rK$  (eqn A.2); cost function  $C(Y_j) = Y_j \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha}$  (eqn A.3); and constant marginal cost of production  $\frac{\partial C}{\partial Y_j} = \frac{\partial C}{\partial q_{ij}} = \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{r}{1-\alpha}\right)^{1-\alpha}$  (eqn A.4).<sup>28</sup>

Substituting the optimal input choices and the associated cost function into the original optimization problem [PF] reduces the dimension of the problem to one in order flows  $\{q_{ij}\}$  only:

#### [Optimization Problem PF\*]

$$W\left(Q,w\right) = \max_{\left\{q_{ij}\right\}_{i \in I, j \in J}} \left\{ v \int_{0}^{1} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} \right] di - \sum_{j \in J} \left[ \left( \int_{0}^{1} q_{ij} di \right) \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1 - \alpha} \right)^{1 - \alpha} \right] \right\}$$
s.t. 
$$\sum_{j \in J} \frac{1}{f_{ij}} q_{ij} \leq Q_{i} \quad \forall i \in [0, 1] \quad \text{[Demand cap]}$$

$$q_{ij} \geq 0 \quad \forall i \in [0, 1], j \in J \quad \text{[Non-negative inputs]}$$

We can set up the Kuhn Tucker Lagrangian for Problem PF\* as:

$$\mathscr{L}^{PF} = v \int_{0}^{1} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} \right] di - \sum_{j \in J} \left[ \left( \int_{0}^{1} q_{ij} di \right) \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1 - \alpha} \right)^{1 - \alpha} \right] - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di - \sum_{i \in I} \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] di -$$

where by symmetry we have  $Q_i = Q$ ,  $\forall i \in [0,1]$ .

The first-order conditions (FOCs) with the corresponding complementary slackness conditions are given by:

The first equality  $\frac{\partial C}{\partial Y_j} = \frac{\partial C}{\partial q_{ij}}$  holds when the "feasibility of intermediate goods order flow" binds with equality in equilibrium.

$$q_{ij}^{PF} \frac{\partial \mathcal{L}^{PF}}{\partial q_{ij}} = q_{ij} \left( \frac{v}{f_{ij}} - \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1 - \alpha} \right)^{1 - \alpha} - \frac{\lambda_i}{f_{ij}} \right) = 0 \quad \forall i \in I, j \in J \quad (B.1)$$

$$\lambda_{i} \frac{\partial \mathcal{L}^{PF}}{\partial \lambda_{i}} = \lambda_{i} \left[ \sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q \right] = 0 \quad \forall i \in I$$
(B.2)

We observe from the FOCs that for each final good i, the corresponding Lagrangian multiplier  $\lambda_i$ , when strictly positive, is determined by the supplier  $\underline{j} \in J$  that can provide the inputs most cheaply to i:

$$\lambda_{i} = v - \min_{j \in J} \left\{ f_{ij} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1 - \alpha} \right)^{1 - \alpha} \right\}$$

$$= v - f_{i\underline{j}} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1 - \alpha} \right)^{1 - \alpha}$$
(B.3)

where  $\underline{j}(i) \in \underline{J}(i) \coloneqq \left\{ \tilde{j} \in J | f_{i\tilde{j}} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \le f_{ij} \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \quad \forall j \in J \right\}.$  Alternatively, if  $v < \min_{j \in J} \left\{ f_{ij} \left( \frac{w_j}{\alpha_j} \right)^{\alpha_j} \left( \frac{r_j}{1-\alpha_j} \right)^{1-\alpha_j} \right\}$ , then  $q_{ij} = 0$  for all  $j \in J$ ,  $\tilde{Y}_i = 0$  and  $\lambda_i = 0$  (i.e. it is not efficient for firm i to produce at all). This latter case is ruled out by the full production assumption (A2).

Thus, combining equations (B.2, B.3), when  $\lambda_i > 0$  final firm i will be allocated sufficient intermediate goods from its cheapest supplier to meet final demand Q:

$$q_{ij}^{PF} = \begin{cases} \frac{1}{n(\underline{J}(i))} f_{i\underline{j}} Q & \text{for } \underline{j} \in \underline{J}(i) \\ 0 & \text{for } j \neq \underline{J}(i) \end{cases}$$

where  $n(\underline{J}(i))$  is the cardinality of the set  $\underline{J}(i)$ . In a symmetric equilibrium, the cheapest supplier(s) coincides with the closest supplier(s). With intermediate good firms located equidistant around the circle, there are at most two closest suppliers for each i (e.g. nodes 0 and 1 for  $i = \frac{1}{2n}$ ). In such cases when there are two closest suppliers, instead of tie-breaking by dividing order volumes in half, we assume each intermediate good node j wins the tie-break to its right on the circle, but loses the

tie-break to its left. This is loosely equivalent to imposing  $n(\underline{J}(i)) = 1, \forall i \in [0, 1]$ ; a convention we will adopt to simplify exposition without loss of generality.

Having solved for the optimal order flow  $\left\{q_{ij}^{PF}\right\}$ , we can now derive the aggregate output of intermediate goods. By symmetry every intermediate good firm j will produce the same amount  $Y_j = Y^{PF}$ ,  $\forall j \in J$ . So we can compute  $Y^{PF}$  from the perspective of firm j = 0, who is able to capture the two equal market segments to its left and right-hand side,  $i \in \left[0, \frac{1}{2n}\right]$  and  $\left[1 - \frac{1}{2n}, 1\right]$ :

$$Y^{PF}(Q,w) = \int_0^1 q_{i0}^{PF} di = 2Q \int_0^{\frac{1}{2n}} f_{i0} di$$
 (B.4)

Finally, we can substitute the equilibrium intermediate good production  $Y^{PF}$  into the Cobb-Douglas production function, combined with the optimality condition for inputs (eqn A.2) to derive explicit solutions for  $K^{PF}$  and  $L^{PF}$ :

$$K^{PF}(Q, w) = \left(\frac{w}{r} \frac{(1-\alpha)}{\alpha}\right)^{\alpha} \left[2Q \int_{0}^{\frac{1}{2n}} f_{i0} di\right]$$
$$L^{PF}(Q, w) = \left(\frac{r}{w} \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left[2Q \int_{0}^{\frac{1}{2n}} f_{i0} di\right]$$

This completes the proof for Proposition 1.

# C Proof for Proposition 2 - Social Planner's Constrained Optimal problem

## C.1 Period 1 optimization

Taking a similar approach to the Perfect Foresight benchmark, we start by forming the corresponding Kuhn Tucker Lagrangian for the social planner's constrained

optimal problem in period 1 [SP1]:

$$\mathcal{L} = \max_{\left\{q_{ij} \in \mathbb{R}_{+}\right\}_{i,j}} v \int_{0}^{1} \left(\sum_{j \in J} \frac{1}{f_{ij}} q_{ij}\right) di - \sum_{j \in J} \left[rK + w_{j} \left(\int_{0}^{1} q_{ij} di\right)^{\frac{1}{\alpha}} K^{-\frac{(1-\alpha)}{\alpha}}\right] \dots$$
$$- \int_{0}^{1} \lambda_{i} \left(\sum_{j \in J} \frac{1}{f_{ij}} q_{ij} - Q\right) di$$

From the Lagrangian we obtain the first-order derivatives with complementary slackness conditions:

$$q_{ij}\frac{\partial \mathcal{L}}{\partial q_{ij}} = q_{ij}\left(\frac{v}{f_{ij}} - \frac{w}{\alpha}\left(\frac{\int_0^1 q_{ij}di}{K}\right)^{\frac{1-\alpha}{\alpha}} - \frac{\lambda_i}{f_{ij}}\right) = 0 \quad \forall i \in [0,1], \forall j \in J$$

$$\lambda_i \frac{\partial \mathcal{L}}{\partial \lambda_i} = \lambda_i \left(\sum_{i \in J} \frac{1}{f_{ij}} q_{ij} - Q\right) = 0 \quad \forall i \in [0,1]$$

where  $\frac{v}{f_{ij}}$  is the marginal benefit from supplying i from j (i.e.,  $q_{ij}$ ), and  $\frac{w}{\alpha} \left( \frac{\int_0^1 q_{ij} di}{K} \right)^{\frac{1-\alpha}{\alpha}}$  is the marginal cost. Later, in the final step of this proof, we will substitute out the endogenously determined K and  $q_{ij}$  to show that  $\frac{w}{\alpha} \left( \frac{\int_0^1 q_{ij} di}{K} \right)^{\frac{1-\alpha}{\alpha}} = \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \left( \frac{wQ^{\frac{1}{\alpha}}}{E \left[ wQ^{\frac{1}{\alpha}} \right]} \right)^{1-\alpha}$ .

By the full-production assumption, we know that the marginal benefit will always weakly exceed the marginal cost, so the Lagrangian multiplier for final good firm i,  $\lambda_i$ , is given by the intermediate good firm  $\underline{j}$  that offers the lowest effective cost:

$$\lambda_{i} = v - \min_{j \in J} \left\{ f_{ij} \frac{w}{\alpha} \left( \frac{\int_{0}^{1} q_{ij} di}{K} \right)^{\frac{1-\alpha}{\alpha}} \right\}$$
$$= v - f_{i\underline{j}} \frac{w}{\alpha} \left( \frac{Y}{K} \right)^{\frac{1-\alpha}{\alpha}}$$

Defining the set of lowest effective cost suppliers as:

$$\underline{J}(i) := \left\{ \tilde{j} \in J | f_{i\tilde{j}} \frac{w}{\alpha} \left( \frac{\int_0^1 q_{ij} di}{K} \right)^{\frac{1-\alpha}{\alpha}} \leq f_{ij} \frac{w}{\alpha} \left( \frac{\int_0^1 q_{ij} di}{K} \right)^{\frac{1-\alpha}{\alpha}} \quad \forall j \in J \right\}$$

we arrive at the first part of the proposition (eqn 3.14):

$$q_{ij}^{SP}(Q, w) = \begin{cases} f_{ij}Q & \text{if } j \in \underline{J}(i) \text{ and } v \ge f_{ij}\frac{w}{\alpha} \left(\frac{Y^{SP}}{K^{SP}}\right)^{\frac{1-\alpha}{\alpha}} \\ 0 & \text{otherwise} \end{cases}$$

where 
$$Y^{SP} = \int_0^1 q_{ij}^{SP} di$$

Next, from the cost-minimization problem for the Cobb-Douglas production with partial delay (eqn A.5) we have the next part of the proposition for the optimal choice of the scalable input factor in period 1:

$$L^{SP}\left(Q,w\right)=Y^{\frac{1}{\alpha}}K^{-\frac{\left(1-\alpha\right)}{\alpha}}=\left(\int_{0}^{1}q_{ij}^{SP}di\right)^{\frac{1}{\alpha}}\left(K^{SP}\right)^{-\frac{\left(1-\alpha\right)}{\alpha}}$$

## C.2 Period 0 Optimization

As discussed in the main body, we can show that the optimality condition for non-scalable production capacity  $K^{SP}$  (equation 3.16) holds with or without the full production assumption. To elucidate this point, note that when the full production assumption is relaxed, there may exist states of the world (Q, w) where some final good firms situated far from intermediate good production firms do not find it optimal to produce at all. In other words, let  $\bar{i}_0^{SP}\left(\tilde{Q},\tilde{w}\right)$  represent a "threshold" firm in the final goods sector. This firm is indifferent between sourcing inputs from intermediate good firm j=0 and opting out of production altogether in state  $(\tilde{Q},\tilde{w})$ :  $\frac{v}{f(\bar{i}_0^{SP})}=\left(\frac{\tilde{w}}{\alpha}\right)^{\alpha}\left(\frac{r}{1-\alpha}\right)^{1-\alpha}\left(\frac{\tilde{w}\tilde{Q}^{\frac{1}{\alpha}}}{E\left[wQ^{\frac{1}{\alpha}}\right]}\right)^{1-\alpha}$ . Hence there may exist states  $(\tilde{Q},\tilde{w})$  whereby  $\bar{i}_0^{SP}\left(\tilde{Q},\tilde{w}\right)<\frac{1}{2n}$ , and the market segment  $\left[\bar{i}_0^{SP}\left(\tilde{Q},\tilde{w}\right),\frac{1}{2n}\right]$  on the circle produces no final good outputs and experiences "empty shelves". At first glance, one might expect that an ex-ante increase in non-scalable capacity K would

positively impact welfare. This expectation arises from the fact that an increase in K would endogenously boost the production of intermediate goods, Y. However, the indirect effects captured by  $\frac{dY}{dK}$  are zero in equilibrium. We can safely ignore the indirect effects of K on Y, because the indirect effects are multiplied by the difference between the marginal cost and marginal benefit of production for the threshold buyer, which is equal to zero by construction. Therefore, irrespective of whether the full-production assumption holds, only the direct effects of K matter.

Formally, recall that the period 1 value function for given K and realization of Q and w can be expressed as the difference between the value of final goods produced and the cost of the required intermediary goods:

$$W^{SP}(K|Q,w) = v \left(2n \int_0^{\min\left\{\frac{1}{2n},\vec{t}_0^{SP}\right\}} Qdi\right) - n \left(rK + wY^{\frac{1}{\alpha}}K^{-\frac{(1-\alpha)}{\alpha}}\right)$$
 (C.1)

where  $i_0^{SP}$  is the *threshold buyer* for intermediate good 0, (implicitly) defined as the final good firm i for which the marginal benefit of sourcing inputs from j = 0 equals the marginal cost:

$$\frac{v}{f_{\tilde{t}_0^{SP},0}} = \frac{w}{\alpha} \left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}} \tag{C.2}$$

The upper limit of integration,  $\min\left\{\frac{1}{2n}, \bar{i}_0^{SP}\right\}$ , reflects the possibility of "regime switching" when the full production assumption is relaxed. When the economy operates at a full-production equilibrium, the relevant threshold buyer for intermediate good firm j=0 is given by  $i=\frac{1}{2n}$ , the final good firm located at the half way point between j=0 and j=1. This is a *competitive regime*, where intermediate good firms engage in monopolistic competition. But without the full-production assumption, there may arise states of the world whereby the threshold buyer for j=0 is closer: i.e.  $\bar{i}_0^{SP} < \frac{1}{2n}$ . This is a *local monopolies* regime, characterized by a gap in market coverage between two supplier nodes (e.g. between j=0 and j=1). The

<sup>&</sup>lt;sup>29</sup>This result bears resemblance to the Envelope Theorem, in which the total derivative of the value function with respect to the parameters of the model is equal to its partial derivative. Here K is the choice variable, but the total derivative of  $W^{SP}(K|Q,w)$  with respect to K is also equal to its partial derivative.

demand for final goods is not fully met for firms located in this gap, and we see "empty shelves" in some segments of the market. We account for the possibility of "regime switching" between the competitive regime and the local monopolies regime in the analyses that follows.

Totally differentiating the expectation of  $W^{SP}$  with respect to K will yield the desired first-order optimality condition for non-scalable capacity in period 0. For ease of exposition, we proceed with the differentiation in parts. In particular, note that the change in the final output in each market segment with respect to K is given by:

$$rac{d}{dK}\int_0^{\min\left\{rac{1}{2n},ec{i}_0^{SP}
ight\}}Qdi=rac{d\min\left\{rac{1}{2n},ec{i}_0^{SP}
ight\}}{dK}Q$$

which depends on the derivative of the threshold buyer  $\bar{i}_0^{SP}$  with respect to K. Strictly speaking, the function  $\min\left\{\frac{1}{2n}, \bar{i}_0^{SP}\right\}$  is not continuously differentiable w.r.t. K due to the kink where  $\frac{1}{2n} = \bar{i}_0^{SP}$ . Without loss of generality, we will loosely define  $\frac{d\min\left\{\frac{1}{2n}, \bar{i}_0^{SP}\right\}}{dK}$  using its right-hand side derivative:

$$\frac{d\min\left\{\frac{1}{2n}, \bar{i}_{0}^{SP}\right\}}{dK} = \begin{cases} 0 & \text{when } \min\left\{\frac{1}{2n}, \bar{i}_{0}^{SP}\right\} = \frac{1}{2n} \\ \frac{d\bar{i}_{0}^{SP}}{dK} > 0 & \text{when } \min\left\{\frac{1}{2n}, \bar{i}_{0}^{SP}\right\} < \frac{1}{2n} \end{cases}$$

to account for the fact that when  $\bar{i}_0^{SP} \ge \frac{1}{2n}$ , the presence of demand caps in overlapping market segments means that any further increases in capacity would not increase aggregate output.

Next, the increase in scalable input costs (wL) from changes in K can be broken down into two components: the indirect costs of requiring more scalable inputs when total output increase following a rise in K; minus the direct cost savings of needing less L when K increases for given output Y:

$$\frac{d}{dK}\left(wY^{\frac{1}{\alpha}}K^{-\frac{(1-\alpha)}{\alpha}}\right) = w\left[\frac{1}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\frac{dY}{dK} - \frac{1-\alpha}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}}\right]$$

Since  $Y = 2Q \int_0^{\min\left\{\frac{1}{2n}, \bar{t}_0^{SP}\right\}} f_{i0} di$ , the endogenous increase in increase in output Y

when *K* increase is:

$$\begin{split} \frac{dY}{dK} &= 2Q \left[ \frac{d \min\left\{ \frac{1}{2n}, \bar{t}_{0}^{SP} \right\}}{dK} f_{\min\left\{ \frac{1}{2n}, \bar{t}_{0}^{SP} \right\}, 0} \right] \\ &= 2Q \left[ \frac{d \min\left\{ \frac{1}{2n}, \bar{t}_{0}^{SP} \right\}}{dK} f_{\bar{t}_{0}^{SP}, 0} \right] \end{split}$$

The second equality simplifies the first by noting that  $\frac{d \min\left\{\frac{1}{2n}, \overline{t_0^{SPS}}\right\}}{dK} = 0 \Leftrightarrow \min\left\{\frac{1}{2n}, \overline{t_0^{SPS}}\right\} = \frac{1}{2n}$ .

Taken together, we have the following first-order optimality condition for the period 0 problem:

$$\frac{dE\left[W^{SP}\right]}{dK} = 2nvE\left[\frac{d\min\left\{\frac{1}{2n},\bar{i}_{0}\right\}}{dK}Q\right] - nr - nE\left[w\left(\frac{1}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\frac{dY}{dK} - \frac{1-\alpha}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}}\right)\right] = 0$$

$$\Leftrightarrow 0 = 2vE\left[\frac{d\min\left\{\frac{1}{2n},\bar{i}_{0}\right\}}{dK}Q\right] - r - E\left[w\left(\frac{1}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\frac{dY}{dK} - \frac{1-\alpha}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}}\right)\right]$$

$$\Leftrightarrow 0 = vE\left[\frac{1}{f_{\bar{i}^{SP},0}}\frac{dY}{dK}\right] - r - E\left[\frac{w}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\frac{dY}{dK}\right] + \frac{1-\alpha}{\alpha}E\left[w\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}}\right]$$

$$\Leftrightarrow r = E\left[\left(\frac{v}{f_{\bar{i}^{SP},0}} - \frac{w}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\right)\frac{dY}{dK}\right] + \frac{1-\alpha}{\alpha}E\left[w\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}}\right]$$

$$\Leftrightarrow r = E\left[0 \times \frac{dY}{dK}\right] + \frac{1-\alpha}{\alpha}E\left[\frac{wL}{K}\right]$$

$$\Leftrightarrow \alpha rK^{SP} = (1-\alpha)E\left[wL^{SP}\right]$$

where the penultimate line holds because  $\frac{v}{f_{\tilde{t}_0^{SP},0}} = \frac{w}{\alpha} \left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}$  (marginal benefit = marginal cost) is the definition of  $\bar{t}_0^{SP}$ , and from the Cobb-Douglas production function, we have  $\left(\frac{Y}{K}\right)^{\frac{1}{\alpha}} = \frac{L}{K}$ . In other words, the indirect effects of raising K on aggregate output Y neatly cancels out, leaving us with the familiar Cobb-Douglas inputs optimality condition in the final line.

Using the production function to substitute out  $L^{SP} = Y^{\frac{1}{\alpha_j}} K^{-\frac{(1-\alpha_j)}{\alpha_j}}$  and rearranging yields the explicit solution for  $K^{SP}$ :

$$K^{SP} = \left(rac{1-lpha}{lpha}rac{1}{r}
ight)^{lpha} \left(E\left[w\left(2Q\int_{0}^{\min\left\{rac{1}{2n},ar{l}_{0}^{SP}
ight\}}f_{i0}di
ight)^{rac{1}{lpha}}
ight]
ight)^{lpha}$$

as required for part 2 of the proposition.

Finally, to complete the proof, we want to show that this level of  $K^{SP}$  indeed leads to a full production equilibrium under assumption A2. We do this by substituting out the explicit expression for  $K^{SP}$  in the marginal cost function to show that in equilibrium the marginal cost of production is always below the valuation for the final good (adjusted for the distance-based penalty):

$$\widetilde{MC}(Q, w) := \frac{w}{\alpha} \left( \frac{Y^{SP}}{K^{SP}} \right)^{\frac{1-\alpha}{\alpha}}$$

$$= \frac{w}{\alpha} \left( r \frac{\alpha}{(1-\alpha)} \right)^{1-\alpha} \left( \frac{Y^{SP}}{E \left[ w(Y^{SP})^{\frac{1}{\alpha}} \right]^{\alpha}} \right)^{\frac{1-\alpha}{\alpha}}$$

$$= \left( \frac{w}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \left( \frac{wQ^{\frac{1}{\alpha}}}{E \left[ wQ^{\frac{1}{\alpha}} \right]} \right)^{1-\alpha}$$

$$\leq \left( \frac{\overline{w}}{\alpha} \right)^{\alpha} \left( \frac{r}{1-\alpha} \right)^{1-\alpha} \left( \frac{\overline{w}Q^{\frac{1}{\alpha}}}{E \left[ wQ^{\frac{1}{\alpha}} \right]} \right)^{1-\alpha} \quad \forall w, Q$$

$$\leq \frac{v}{f\left( \frac{1}{2n} \right)} \quad \text{by assumption A2}$$

where  $Y^{SP}(Q, w) = 2Q \int_0^{\frac{1}{2n}} f(i) di$ .

This completes the proof for Proposition 2.

## C.3 Relationship between $K^{PF}$ and $K^{SP}$

Recall from equations 3.9, 3.17 that we have:

$$K^{PF}(Q, w) = \left(\frac{w}{r} \frac{(1 - \alpha)}{\alpha}\right)^{\alpha} \left(2Q \int_{0}^{\frac{1}{2n}} f(i) di\right)$$
$$K^{SP} = \left(\frac{1}{r} \frac{1 - \alpha}{\alpha}\right)^{\alpha} \left(2 \int_{0}^{\frac{1}{2n}} f(i) di\right) \left(E\left[wQ^{\frac{1}{\alpha}}\right]\right)^{\alpha}$$

Some straight-forward algebra shows that:

$$K^{SP} = K^{PF} (Q, w) \left( \frac{E \left[ wQ^{\frac{1}{\alpha}} \right]}{wQ^{\frac{1}{\alpha}}} \right)^{\alpha}$$

Furthermore, taking the expectation of  $K^{PF}$  over (Q, w), we have

$$E\left[K^{PF}\left(Q,w\right)\right] = \left(\frac{1}{r}\frac{\left(1-\alpha\right)}{\alpha}\right)^{\alpha} \left(2\int_{0}^{\frac{1}{2n}}f\left(i\right)di\right)E\left[w^{\alpha}Q\right]$$

Taken together with the expression for  $K^{SP}$ , we can show:

$$\frac{K^{SP}}{E\left[K^{PF}\left(Q,w\right)\right]} = \frac{\left(E\left[wQ^{\frac{1}{\alpha}}\right]\right)^{\alpha}}{E\left[w^{\alpha}Q\right]}$$

such that, by Jensen's inequality and given  $g(x) := x^{\alpha}$  is concave for  $\alpha \in (0,1)$ , we have:

$$K^{SP} \ge E\left[K^{PF}\left(Q,w\right)\right]$$

as required.

## D Sufficient condition for full production symmetric equilibrium in the decentralized solution

First, we establish the sufficient conditions for the existence of a full-production symmetric equilibrium.

**Lemma 1.** [Existence of Full Production Symmetric Equilibrium]: For every configuration of the primitives of the model with the exception of v,  $\mathscr{E}_{-v} = \{f(\cdot), \alpha, w, r, Q\}$ , there exist a  $\bar{v} \in \mathbb{R}_{++}$  such that the economies  $\mathscr{E}(v) = \{f(\cdot), \alpha, w, r, Q, v \geq \bar{v}\}$  admits a full production symmetric equilibrium.

Intuitively, the marginal benefit of production is increasing in the valuation of the final good v, but the marginal cost is non-increasing in v. So, for every parameterization of the model, we can find a large enough  $\bar{v}$  to guarantee full production in a symmetric equilibrium.

Formally, while assumption A2 establishes the sufficient conditions for full production under the social planner benchmarks, the corresponding full-production condition for the decentralized case is given by:

$$v \ge f\left(\frac{1}{2n}\right)p^* = f\left(\frac{1}{2n}\right)\mu\left(n\right)\widetilde{MC} = f\left(\frac{1}{2n}\right)\mu\left(n\right)\frac{\bar{w}}{\alpha}\left(\frac{\bar{Q}\int_0^{\frac{1}{2n}}f\left(i\right)di}{K^*}\right)^{\frac{1-\alpha}{\alpha}}$$

where  $p^*$  is the equilibrium price for intermediate goods,  $\mu\left(n\right) := \frac{f\left(\frac{1}{2n}\right)}{f\left(\frac{1}{2n}\right) - 2\frac{f'\left(\frac{1}{2n}\right)}{f\left(\frac{1}{2n}\right)} \int_0^{\frac{1}{2n}} f(i)di}$ 

is the mark-up over marginal costs, and  $K^*$  is the equilibrium level of non-scalable capacity. We argue that for every possible parameterization of the other primitives, there exists a  $\bar{v} \in \mathbb{R}_{++}$  that guarantees full production.

Consider an arbitrary economy  $\mathscr{E}(\tilde{v}) = \{f(\cdot), \alpha, w, r, Q; \tilde{v}\}$  with valuation  $\tilde{v}$ . We want to show that by varying  $\tilde{v}$  we can always construct an economy  $\mathscr{E}(\bar{v}) = \{f(\cdot), \alpha, w, r, Q; \bar{v}\}$  that supports a full production symmetric equilibrium holding all other primitives the same. To do this, we compute  $K^*(\tilde{v})$ , the associated equilibrium level of capacity investment *assuming* full production; and the correspond-

ing  $\overline{MC}(\tilde{v}) = f\left(\frac{1}{2n}\right)\mu\left(n\right)\frac{\bar{w}}{\alpha}\left(\frac{\bar{Q}\int_0^{\frac{1}{2n}}f(i)di}{K^*(\bar{v})}\right)^{\frac{1-\alpha}{\alpha}}$ , the highest possible realization of marginal costs in that economy. Note that  $K^*\left(v\right)$  is a non-decreasing function of v and therefore  $\overline{MC}\left(v\right)$  is a non-increasing function of v (i.e. the marginal cost of production in any full production equilibrium does not increase when the valuation increases). Then if  $\tilde{v} \geq \overline{MC}\left(\tilde{v}\right)$ , then the economy  $\mathscr{E}\left(\tilde{v}\right)$  admits a full production symmetric equilibrium characterized by  $K^*\left(\tilde{v}\right)$ . If instead  $\tilde{v} < \overline{MC}\left(\tilde{v}\right)$ , let  $\bar{v} = \overline{MC}\left(\tilde{v}\right) > \tilde{v}$ . Then  $\bar{v} = \overline{MC}\left(\tilde{v}\right) \geq \overline{MC}\left(\bar{v}\right)$ . And every economy  $\mathscr{E}\left(v\right) = \{f\left(\cdot\right), \alpha, w, r, Q; v \geq \bar{v}\}$  admits a full production symmetric equilibrium as required.

Second, we remark that the full production assumption also enables us to avoid problems of non-differentiability in the demand function. In a classical treatment of the circular economy, Salop (1979) segments the demand function for intermediate goods into three sections: a "monopoly" regime (whereby the firm acts as if it is a monopoly); a "competitive" regime (where it engages in Bertrand competition with its neighbors); and a "super-competitive" regime (where it prices so aggressively as to take over its neighbor's native market). The demand function exhibits a kink at the intersection between the monopoly and competitive regime, and makes a discontinuous jump between the competitive and super-competitive regime. We can rule out equilibria falling under the super-competitive regime by setting a sufficiently steep distance-based penalty function; and for the purpose of the main analyses in section 3 and 4, the full production assumption ensures the demand function is continuously differentiable. In Appendix G we relax the full production assumption to examine the interplay between the competitive and monopoly regime.

# E Proof of Proposition 3: Full production symmetric equilibrium in the spot market

In period 1, the equilibrium spot market orders  $q_{ij}^{spot,*}$  by final good firms, and the purchase of scalable inputs  $L^*$  by intermediate good firms, take a similar form to their corresponding expressions under the constrained optimal benchmark. We skip their derivations to avoid repetition, and concentrate instead on the solution for the

spot market price  $p^*$ , given by equation 4.10.

To solve for  $p^*$ , we will first need to derive the demand function facing the intermediate good firm j=0 on the spot market. For now, we will also need to conjecture that the aggregate volume of pre-orders must be equalized across all final good firms in equilibrium:  $Q_i^{pre} = Q^{pre}$ ,  $\forall i \in I$ , a result that we will prove formally later in appendix F.

### **E.1** Finding the slope of the demand curve

The demand curve facing each intermediate good firm is piece-wise linear (when plotted against  $p_j$ , for given  $\mathbf{p}_{-j}$ ). To see this, note that the period 1 equilibrium is governed by two indifference thresholds. First, for given price vector  $(p_0, \mathbf{p}_{-0})$ , the *participation threshold* for firm j=0,  $\bar{i}_0$ , is defined as the final good firm that is indifferent between buying inputs from intermediate good firm j=0 and not producing at all:

$$f(\bar{i}_0) p_0 = v, \quad \forall p_0 \in \left[\frac{v}{f(\frac{1}{2})}, v\right]$$
 (E.1)

Second, the *competitive threshold*  $\bar{i}_{0,1}$  is the marginal final good producer that is indifferent from buying from supplier node j = 0 and j = 1:

$$f(d(\bar{i}_{0,1},0)) p_0 = f(d(\bar{i}_{0,1},1)) p_{-0}$$
 (E.2)

Hence the demand curve facing firm j = 0 depends on the lower envelope of the participation and competitive threshold functions:

$$Y_0^{spot} = 2 \int_0^{\bar{t}_0^*} f(i) \cdot (Q - Q^{pre}) di$$
 (E.3)

where

$$\bar{i}_{0}^{*} := \min \left\{ \bar{i}_{0} \left( p_{0} \right), \bar{i}_{0,1} \left( p_{0}, p_{-0} \right) \right\}$$
 (E.4)

When the slope of the demand curve is well-defined (i.e., away from the knife-

edge case when  $\bar{i}_0(p_0) = \bar{i}_{0,1}(p_0, p_{-0})$ ), it is given by:

$$\frac{dY_0^{spot}}{dp_0} = 2 \left[ \frac{d\bar{i}_0^*}{dp_0} f(\bar{i}_0^*) (Q - Q^{pre}) \right]$$
 (E.5)

Under a full production symmetric equilibrium we have  $\bar{i}_0^* = \bar{i}_{0,1} (p^*, p^*) = \frac{1}{2n}$ , and

$$\frac{d\bar{i}_{0}^{*}(p_{0}=p^{*},p_{-0}=p^{*})}{dp_{0}} = \frac{d\bar{i}_{0,1}(p^{*},p^{*})}{dp_{0}} = -\frac{\partial\left(\frac{f(\bar{i}_{0,1})}{f(\frac{1}{n}-\bar{i}_{0,1})}p_{0}\right)/\partial p_{0}}{\partial\left(\frac{f(\bar{i}_{0,1})}{f(\frac{1}{n}-\bar{i}_{0,1})}p_{0}\right)/\partial \bar{i}_{0,1}}$$

$$= -\frac{1}{\left(\frac{f'(\bar{i}_{0,1})}{f(\bar{i}_{0,1})} + \frac{f'(\frac{1}{n}-\bar{i}_{0,1})}{f(\frac{1}{n}-\bar{i}_{0,1})}\right)p_{0}}$$

$$= -\frac{f\left(\frac{1}{2n}\right)}{2f'\left(\frac{1}{2n}\right)p_{0}} \tag{E.6}$$

#### **E.2** Solving for the optimal spot market price

We can derive the following first-order condition with respect to  $p_0$  from the intermediate good producer j = 0's optimization problem (equation 4.5):

$$\left(p^* - \frac{w}{\alpha} \left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}\right) = Y^{spot} / \left(-\frac{dY^{spot}}{dp^*}\right)$$
 (E.7)

where  $\frac{w}{\alpha}\left(\frac{Y}{K}\right)^{\frac{1-\alpha}{\alpha}}$  is the marginal cost of production for intermediate goods;  $Y:=Y^{spot}+Y^{pre}$  is the total amount of intermediate good production; and  $\frac{dY^{spot}}{dp^*}$  is the slope of the demand curve in the spot market. By imposing symmetry we get  $p^*=p_0^*=p_j^*$  for all  $j\in J$ .

Substituting equations E.5 and E.6 into equation E.7 gives the optimal spot-

market price as required:

$$\begin{split} (p^* - MC) &= \frac{2 \int_0^{\frac{1}{2n}} f(i) (Q - Q^{pre}) di}{-2 \left[ -\frac{f(\frac{1}{2n})}{2f'(\frac{1}{2n})p^*} f(\frac{1}{2n}) (Q - Q^{pre}) \right]} \\ \Rightarrow p^* &= \left( 1 + \frac{2f'(\frac{1}{2n}) \int_0^{\frac{1}{2n}} f(i) di}{\left( f(\frac{1}{2n}) \right)^2 - 2f'(\frac{1}{2n}) \int_0^{\frac{1}{2n}} f(i) di} \right) MC \end{split}$$

# F Proof of proposition 4: Full Production Symmetric Equilibrium in the pre-order market

#### F.1 Final good producers in period 0

We will start by verifying that the conjecture  $Q_i^{pre} = Q^{pre}$ ,  $\forall i \in I$  is indeed an equilibrium solution.

**Lemma 2.** [Optimal Pre-orders] In a full-production symmetric equilibrium, each final good producer i will:

1. pre-order from the intermediate good producers that sets the lowest effectiveprice for i.

$$q_{ij}^{pre,*} = \begin{cases} f_{ij}Q_i^{pre,*} & \text{if } j \in \underline{J}(i;\phi), \text{ and } f_{ij}\phi_j \leq v \\ 0 & \text{otherwise} \end{cases}$$
(F.1)

where  $\underline{J}(i;\phi) := \left\{ \tilde{j} \in J : f_{i\tilde{j}}\phi_{\tilde{j}} = \min \left\{ \mathbf{f}_i \circ \phi \right\} \right\}$  denote the set of suppliers that provides the lowest effective price for i.

2. set the aggregate quantity of pre-orders  $Q_i^{pre,*}$  such that the marginal cost of pre-orders is equal to its expected marginal benefit.

$$f_{i\tilde{j}}\phi_{\tilde{j}} = \Pr\left(Q > Q_{i}^{pre,*}\right) E\left[f_{i\hat{j}}p_{\hat{j}}^{*}(Q,w) | Q > Q_{i}^{pre,*}\right], \quad for \ \tilde{j} \in \underline{J}(i;\phi), \hat{j} \in \underline{J}(i;\mathbf{p})$$

$$(F.2)$$

$$where \ \underline{J}(i;\mathbf{p}) := \left\{\hat{j} \in J : f_{i\hat{j}}p_{\hat{j}} = \min\left\{\mathbf{f}_{i} \circ \mathbf{p}\right\}\right\}$$

Furthermore, imposing symmetry implies

$$\phi = \Pr\left(Q > Q_i^{pre,*}\right) E\left[p^*\left(Q,w\right) | Q > Q_i^{pre,*}\right] \quad \forall i \in [0,1]$$
 (F.3)

so that the aggregate volume of pre-orders must be equalized across all final good firms:

$$Q_i^{pre,*} = Q^{pre,*} \quad \forall i \in [0,1]$$
 (F.4)

Equation F.2 is the first-order condition of final good producer i's period 0 optimization problem. It gives an implicit expression for the equilibrium aggregate volume of pre-orders  $Q_i^{pre,*}$  as a function of spot and pre-order prices  $(p^*, \phi)$ . On the left hand side of the equation,  $f_{i\tilde{i}}\phi_{\tilde{i}}$  is the effective marginal cost of preorders. On the right hand side is the expected marginal benefit of pre-orders, which is equal to the probability that the spot market order of i will be strictly positive  $\Pr\left(Q>Q_i^{pre,*}\right)$ , multiplied by the conditional expectation of the lowest effective spot price, given *i*'s spot-market order is strictly positive  $E\left[f_{i\hat{j}}p^*\left(Q,w\right)|Q>Q_i^{pre,*}\right]$ . Under symmetry,  $p_i^* = p^*$  and  $\phi_j = \phi$  for all  $j \in J$ ; so the nearest intermediate good node to i will always provide the lowest effective price on both the pre-order and spot markets:  $f_{ij} = f_{ij}$ . Equation F.2 can thus be simplified to equation F.3, which we can also interpret as the demand function for pre-orders  $Q_i^{pre,*}$  for given pre-order price  $\phi$ . Equation F.3 has two immediate implications: (1) aggregate preorders must be equalized across i (equation F.4); and (2) the highest sustainable pre-order price is  $\phi = E[p^*]$ , in which case the final good producers will only preorder to satisfy the minimal possible realization of demand  $Q^{pre,*} = Q$ . For any pre-order price greater than the unconditional expectation of the spot market price, the aggregate quantity of pre-order will be zero. So we can view equation F.3 also as a participation constraint for final good firms on the pre-order market.

## **F.2** Intermediate good producers in period 0

Recall the expected profit function for intermediate good producers:

$$\max_{\phi,K} E\left[\Pi_{j}\right] = E\left[p^{*}Y^{spot} - wL^{*}\right] + \phi Y^{pre} - rK$$

$$= \int_{Q^{pre}}^{\bar{Q}} \int_{w} \left(p^{*}Y^{spot}\right) h(w) g(Q) dwdQ \dots$$

$$- \left(\int_{\underline{Q}}^{Q^{pre}} \int_{w} \left(wL^{*}\right) h(w) g(Q) dwdQ + \int_{Q^{pre}}^{\bar{Q}} \int_{w} \left(wL^{*}\right) h(w) g(Q) dwdQ\right) \dots$$

$$+ \phi Y^{pre} - rK \tag{F.5}$$

We note that in a symmetric full production equilibrium, the aggregate production of intermediate goods  $Y := Y^{pre} + Y^{spot} = 2Q \int_0^{\frac{1}{2n}} f(i) di$  is exogenously pinned down by the realization of final good demand Q, and the distance-based penalty function f. But the relative importance of the spot market and the pre-order market  $(Y^{pre} \text{ and } Y^{spot})$  depends on the aggregate volume of pre-orders  $Q^{pre}$ , which is determined by the choice of the pre-order price  $\phi$ . On the other hand, the level of non-scalable capacity investment K affects the period 1 equilibrium spot market price  $p^*(Q, w)$  and scalable input demand  $L^*(Q, w)$  in each possible state. We examine the optimality conditions for  $\phi$  and K in turn.

First we take the derivative of expected profits with respect to  $\phi$ . With some algebra, we can show that

$$\frac{dE\left[\Pi_{j}\left(K_{j},\phi_{j},q_{j}^{pre}\right)\right]}{d\phi} = \phi\left(-\frac{dY^{pre}}{d\phi}\right) - \Pr\left(Q \leq Q^{pre}\right)E\left[w\frac{\partial L^{*}}{\partial Y}|Q \leq Q^{pre}\right]\frac{dY^{pre}}{d\phi} \dots \\
+ \left(Y^{pre} + \phi\frac{dY^{pre}}{d\phi}\right) \qquad (F.6)$$

$$= -\Pr\left(Q \leq Q^{pre}\right)E\left[w\frac{\partial L^{*}}{\partial Y}|Q \leq Q^{pre}\right]\frac{dY^{pre}}{d\phi} + Y^{pre}$$

$$= \Pr\left(Q \leq Q^{pre}\right)E\left[w\frac{\partial L^{*}}{\partial Y}|Q \leq Q^{pre}\right]\left(-\frac{dY^{pre}}{d\phi}\right) + Y^{pre} > 0$$
(F.7)

This imply that the equilibrium must be a corner solution. Intermediate good producers would like to set the highest possible pre-order price subject to the participa-

tion constraint of final good producers (eqn F.3). Thus, from Lemma 2, equilibrium pre-orders will equal to the lowest possible realization of final good demand, and the equilibrium pre-order price will equal the unconditional expectation of the spot market price:

$$Q^{pre,*} = Q (F.8)$$

$$\phi^* = E\left[p^*\left(Q, w\right)\right] \tag{F.9}$$

Next we take the derivative of the expected profit with respect to *K*:

$$E\left[\frac{\partial p^*}{\partial K}Y^{spot}\right] - E\left[w\frac{\partial L^*}{\partial K}\right] - r = 0$$
 (F.10)

where  $L^* = (Y^{pre} + Y^{spot})^{\frac{1}{\alpha}} (K)^{-\frac{1-\alpha}{\alpha}}$ , so

$$\begin{split} \frac{\partial L^*}{\partial K} &= -\left(\frac{1-\alpha}{\alpha}\right) \left(Y^{pre} + Y^{spot}\right)^{\frac{1}{\alpha}} K^{-\frac{1}{\alpha}} \\ &= -\left(\frac{1-\alpha}{\alpha}\right) \frac{L^*}{K} \end{split}$$

Substituting  $\frac{\partial L^*}{\partial K}$  back into the first-order condition to give

$$E\left[\frac{\partial p^*}{\partial K}Y^{spot}\right] + (1 - \alpha)E\left[wL^*\right] = \alpha rK^*$$
 (F.11)

as required.

## **G** Partial Production and Local Monopolies

In this appendix, we discuss the implications of relaxing the full production assumption. Relaxing the assumption allows for shocks that are severe enough to shut out some market segments of final good producers from the spot market. Final good producers that are further away from intermediate good suppliers (i.e., those with less substitutable inputs) will experience greater difficulty adjusting to the shocks.

To see this, note that the period 1 equilibrium is governed by two indifference thresholds (which may or may not be binding). First, for given price vector  $(p_0, p_{-0})$ , where  $p_j = p_{-0} \ \forall j \neq 0$ , the *participation threshold*  $\bar{i}_0$  is defined as the final good firm that is indifferent between buying inputs from intermediate good firm j = 0 and not producing at all:

$$f(\bar{i}_0) p_0 = v, \quad \forall p_0 \in \left[\frac{v}{f(\frac{1}{2})}, v\right]$$
 (G.1)

Second, the *competitive threshold*  $\bar{i}_{0,1}$  is the marginal final good producer that is indifferent between buying from supplier node j = 0 and j = 1:

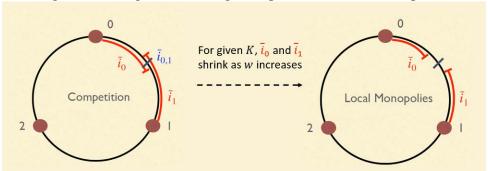
$$f(d(\bar{i}_{0,1},0)) p_0 = f(d(\bar{i}_{0,1},1)) p_{-0}$$
 (G.2)

As Figure G.1 illustrates, the participation threshold  $\bar{i}_0$  (and its counterparts for  $j \neq 0$ ) can be visualized as the arms that reaches out from each supplier node. The participation threshold therefore represents the potential *market reach* for each intermediate good supplier. As long as the market reach from two nearby supplier nodes overlap, the two suppliers engage in competition and the competitive threshold  $\bar{i}_{0,1}$  is the binding threshold for computing demand. Under this *competitive regime*, the intermediate good suppliers' market reach covers every market segment on the circle. The aggregate demand for final goods is met and we see "full shelves". The competitive regime always prevails under the full production assumption.

We can show further that the market reach of each intermediate good supplier is increasing in the level of non-scalable capacity installed (K), and decreasing in the cost of the scalable input (w). For given level of non-scalable capacity K, the market reach of each supplier node gets shorter as the size of the negative cost shock increases, until eventually the participation thresholds  $\bar{i}_0$  and  $\bar{i}_1$  no longer overlap and the two neighboring suppliers (j=0,1) behave like local monopolies. Under this *local monopolies regime*, there is a gap in market coverage between the two supplier nodes, and we see "empty shelves" in some segments of the market.<sup>30</sup>

<sup>&</sup>lt;sup>30</sup>A third possible regime arises when the market reach of one intermediate good supplier goes past the node of another. This is the "super-competitive" regime, whereby one supplier prices so

Figure G.1: Regime switching: competition vs local monopolies



The optimal pricing strategy of intermediate good suppliers therefore depend on whether they are operating under the competitive or the local monopolies regime, which in turn depends on the realization of demand and supply shocks in period 1. We formally characterize the symmetric equilibrium spot-market pricing strategy under the assumption that the distance-based penalty function f(x) takes the form of an exponential function, with parameter  $\beta$ .

**Assumption** A3 Exponential distance-based penalty function:  $f(d) = \exp(\beta d)$ , where  $\beta \in (0,1]$  governs the degree of substitutability between different intermediate goods.

**Proposition 6.** [Optimal spot-market pricing under symmetric equilibrium]

1. Under the competitive regime, we have  $\bar{i}_{0,1} = \frac{1}{2n} \leq \bar{i}_0$  and the equilibrium price for the intermediate good is given by:

$$p_c^* = MC_c \cdot \frac{\exp\left(\frac{\beta}{2n}\right)}{2 - \exp\left(\frac{\beta}{2n}\right)}$$
 (G.3)

aggressively as to capture the home market of their neighboring competitor. Allowing for this possibility would lead to a discontinuous jump in the demand function for intermediate goods. In the interest of tractability, we can rule out the possibility of a super-competitive regime by making the distance-based penalty function  $f(\cdot)$  sufficiently punishing.

where  $MC_c$  is the marginal cost faced by intermediate good suppliers

$$MC_c = \frac{w}{\alpha} \left( \frac{Y^{pre} + Y_c^{spot}}{K} \right)^{\frac{1-\alpha}{\alpha}}$$
 (G.4)

2. Under the local monopolies regime, we have  $\bar{i}_0 < \bar{i}_{0,1} = \frac{1}{2n}$  and the equilibrium price for the intermediate good is given by:

$$p_m^* = \sqrt{v \cdot MC_m} \tag{G.5}$$

where  $MC_m$  is the marginal cost faced by intermediate good suppliers

$$MC_m = \frac{w}{\alpha} \left( \frac{Y^{pre} + Y_m^{spot}}{K} \right)^{\frac{1-\alpha}{\alpha}}$$
 (G.6)

Intuitively, the first part of Proposition 6 shows that under a competitive regime, intermediate goods suppliers charge a mark-up over marginal costs.<sup>31</sup> The mark-up is higher when substitutability is lower (i.e., when  $\beta$ , the parameter governing the distance-based penalty function, is closer to 1), and lower when competition is fiercer (i.e., when n is large). In the limit, as n approaches infinity - and the distance between nodes shrinks to zero such that intermediate goods become perfectly substitutable - equation G.3 simplifies down to the familiar condition of price equals marginal cost.

The second part of Proposition 6 shows that when intermediate good suppliers operate as local monopolies, the price they charge is equal to the geometric average between their marginal costs ( $MC_m$ ) and the highest possible price (v, the valuation of the final good output by end consumers). Unsurprisingly, whilst intermediate good suppliers operates as local monopolies, the number of other firms n is irrelevant to their pricing decision. Any changes in n instead influences whether the economy switches between the local monopolies regime and the competitive regime (i.e. whether  $\bar{i}_0$  is less or greater than  $\bar{i}_{0,1} = \frac{1}{2n}$ ).  $^{32}$ 

<sup>&</sup>lt;sup>31</sup>This part of the proposition is just a re-writing of our earlier results for this specific parameterization.

<sup>&</sup>lt;sup>32</sup>Clearly, this neat characterization of the monopoly price as a geometric average won't hold in

Other factors that influence the market pricing regime that prevails in equilibrium include the level of non-scalable production capacity in place K, and the cost of the scalable input w.

**Proposition 7.** [Regime switching]  $\bar{i}_0$ , the participation threshold (i.e market reach) of firm j = 0, is increasing in K and decreasing in w:

$$\frac{d\bar{i}_0}{dK} > 0 \tag{G.7}$$

$$\frac{d\bar{i}_0}{dw} < 0 \tag{G.8}$$

Proposition 7 formalizes our earlier discussion that, for given non-scalable capacity K, larger negative supply shocks (larger w) increases the likelihood that the economy will end up in the local monopolies regime. Under the local monopolies regime, the market segment ( $i \in (\bar{i}_0, \bar{i}_1)$ ) that lies in-between the market-reach of the two nearby supplier nodes will not be able to fulfill their realized demand for final goods, and we observe "empty shelves". Intuitively, the proposition holds because a higher K, and a lower w, reduces the marginal cost of production, which increases the market reach of the intermediate good supplier.<sup>33</sup>

A key implication of Proposition 7 is that the response of final good outputs to shocks is *non-linear*. Under normal or benign market conditions, the economy might be operating under the competitive regime which ensures that demand from every market segment is met. Market reach of neighboring suppliers overlap, and continues to overlap for small perturbations in supply and demand. Under these benign conditions, the supply network appears robust. But when negative supply shocks becomes sufficiently large, the economy suddenly switches from the competitive regime to the local monopolies regime. The critical role capacity plays, therefore, is that it prevents empty shelves for a larger range of shocks. A larger

general (e.g. without the exponential functional form for f(d)). But the other part of the proposition, that in the local monopolies regime the number of other firms is irrelevant, is more general. Even if other firms exist, they simply aren't selling in each other's "submarket".

 $<sup>^{33}</sup>$ Note that this analysis is not comparative statics in the strict sense: w is an exogenous variable, but K is an endogenous variable. With regard to the latter, we are asking how firms' endogenous choice of capacity investment in period 0 affects market reach and the nature of competition on the spot market in period 1.

*K* allows for a larger market-reach overlap for any given input cost *w*, making the entire network more robust. But since the degree of overlap is in of itself irrelevant, surplus capacity is "wasted" in the absence of large negative supply shocks.

Relaxing the full production assumption therefore reinforces our central message that  $K^* < K^{SP}$ . This is intuitive, because the possibility of a large shock shifting the economy to a local monopolies regime adds another distortion to the system. Ex ante capacity investment K increases network resilience by ensuring full production for a wider range of shocks, but is undervalued by market participants under business-as-usual scenarios. Robustness becomes an externality that may not be fully internalized by individual intermediate good suppliers in their capacity decisions in period 0. Worse still, in imperfectly competitive economies, some firms may profit from the artificial scarcity that arises from a lack of resilience.